Derivation of Dark Matter, Dark Energy and the Cosmological Constant as Features of Space-Time

P. H. Handel[1]* and K. E. Splett[2]†

Affiliations: [1]Department of Physics and Astronomy, University of Missouri-St. Louis & Center for Nanoscience; St. Louis MO 63121 USA. [2]University of Missouri-St. Louis Visiting Scholar 2017-2022; †Present address: Kirschallee 6, Petersroda 06809 Germany. *Corresponding author Emails: phasenoisehandel@gmail.com, erikasplett@yahoo.de; use them jointly

Abstract. This paper reveals the complete solution of the problem of Dark Matter (DM) and Dark Energy (DE), determining their density at any point, disclosing their nature. Based on general relativity (GR) we show that DM is the energy of the coherent gravitational field of the observable universe (OU). We derive simple universal analytic formulae exactly agreeing with current observations, for DM at all levels, the DE density, and the cosmological constant Λ, by anticipating the solution of Einstein's equation for the OU. This yields our "DM key" for DM concentration ρ_D=Qg. Constant Q=1.39·10^{-16} gs^2/cm^4 is the DM concentration accreted per unit local Newtonian gravitational field g, from the OU, on the g of lower-scale structures. We exactly derive the galactic rotation curves, the observed extended galactic halos, Tully-Fisher relations and MOND (generalized). Our key calculates the distribution of DM in galaxy clusters, superclusters, colliding clusters, planetary systems, wide binary stars (see Supplement 1), or on Earth. We derive Λ as 1.08·10^{-56} cm^{-2} and ρ_L=5.8·10^{-30}g/cm^3, with error <3%, from our key, fixed by the Tully-Fisher relations. Supplement 1 explains the true nature of DE and DM, showing in what sense they exist. These results replace previously proposed hypothetical DM models.

Keywords: Astrophysics, Cosmology, Gravitation, Dark matter, Dark Energy, Cosmological Constant, General relativity, · Interdisciplinary physics, · Fundamental 1/f noise

MAIN

Dark Matter as the Gravitational Field Energy of the Observable Universe

According to the well-known ΛCDM (Lambda Cold Dark Matter) model of Astrophysics, about 26% of the mass of the universe is DM,' usually assumed to be some hidden particles, interacting only gravitationally. Ordinary Matter (OM), including gas, radiation, all particles, plasma, dust, stars, planetary systems, galaxies and clusters, represents about 5%, mainly baryonic matter. The rest of 69% is considered[1,2] to be DE and is described in Einstein's equation (EE) by the cosmological constant, observed to be Λ=1.088·10^{-56} cm^{-2}, or by a DE density corresponding to a matter density $\rho_\Lambda = \Lambda c^2/8\pi G$ =5.96·10^{-30} g/cm^3. The DE density ρ_Λ, is also generally considered to be of unknown nature. It is equivalent to a negative pressure that generates a "gravitational repulsion." This is needed to explain with EE the acceleration of the Hubble expansion of the universe, first observed in 1999. The DE is considered responsible for reducing the curvature of the *perceived* universe to zero, causing it to be quite flat, with a curvature parameter Ω_K= 0.0007(19). This was obtained in a 6-Parameter 2018 ΛCDM cosmology fit[2], to the Planck telescope data on the cosmic microwave background temperature, polarization, and lensing. The bracket (19) shows the uncertainty in the last digits of Ω_K. Since the central value of this most recent result is positive, we consider the perceived universe likely to be open and practically flat on the largest scale. We remind that Ω_K<0 corresponds to a finite, spherical geometry of the universe, Ω_K>0 to a hyperbolic open geometry, and Ω_K=0 to a flat (Euclidian) open geometry. The last two cases correspond to an infinite, mostly unobservable, universe.

In a previous paper, we explained DM in principle as energy of the coherent gravitational field of the universe (Eventually divided by c^2 to get it in grams). We defined it as the additional matter needed at all length scales to explain, using classical Newtonian physics, the *observed* gravitational field. *Rather than trying to replace the notion of DM outright, with its expression in terms of the metric tensor, or declaring that it does not exist, we found it reasonable to keep using this re-defined notion of DM.* The gravitational energy of the universe was calculated in four ways[3], both in low-order GR (quadratic in the deviation $h_{\mu\nu}$ from the Minkowski-Euclidian metric), and classically. It was found each time to be of the same order of magnitude as the OM energy. With our new definition, we showed that *DM would largely become its own source[3] in Einstein's Equation, and could indeed self-generate more DM* to be about five times larger than the OM, at the level of the OU. This was shown by using the GR quasi-tensor $t_{\mu\nu}$ of the gravitational field defined by Weinberg pp. 166-167, Eqs. 7.6.14-7.6.15. At the local, lowest scales of the OU, as we will show in the present paper, the simple NA is applicable, with increasing GR corrections at progressively larger scales. These corrections are calculated automatically by our "DM key" algorithm below, and our results agree perfectly with all astrophysical observations. The original derivation, based on a simple electromagnetic analogy, is shown in Methods (M), Sec. 1.

2. The Rotation Speed of Stars in Galaxies

Let us imagine a rigorous solution of the non-linear EE for all coupled levels of the universe, that would define the metric (e.g., the coefficients of terms in Pithagora's theorem) of space-time everywhere, implying the gravitational field and its energy, where the energy represents the DM. The question arises again: *how does the observed DM, about 5 times larger than OM at the level of the OU, manifest itself (in this imagined solution) as gravitational energy in lower-level structures of the universe, i.e. in planetary systems, galaxies, and clusters thereof?*

We would expect the gravitational field of the universe, denoted with the symbol g', to add energy (or DM) with a density dependent on both g and g', to the much smaller gravitational field g of lower-level structures. This smaller g is generated in the local system of reference (just as $g=9.8\ m/s^2$ on earth). Indeed, EE implies a self-interaction of the gravitational field of the OU everywhere, at all levels. This happens because the gravitational field has energy and mass, but at the same time it is also the force acting on each mass unit. Thus, it also acts on its own mass. In the solution of the nonlinear EE for the OU, we therefore expect to find a "universal" energy density contribution ρ_D, which is a function $\rho_D = f(g,g')$ of g and g', in addition to the local Newtonian energy density ρ of the smaller local field g<<g' that is generated in lower-scale structures.

By expanding f(g,g') in series around g=0, with g<<g', we retain the first-order term

$$\rho_D = \rho_D(0) + Qg = Qg, \qquad (1)$$

where Q is a function of g', with $\rho_D(0)=0$. We are interested in the term Qg, which describes the accretion of universal gravitational energy in mass units, per unit volume on the classical local gravitational field g, measured in the rest frame of the lower-level structure considered, e.g., a cluster of galaxies, a galaxy, a star, or a planet. It does not matter where the galaxy is in its cluster, where the star is in the galaxy, or where the planet is in the planetary system, as long as they are in free motion, in "free fall."

The result in Eq. (1) can also be understood on a simple physical basis. Considering g' as a measure of the gravitational field of the universe, g'^2 can symbolize a measure of the energy or mass density of the gravitational field of the universe, i.e. density of DM. *The field g of a lower-scale structure is the force acting on any mass unit* and will therefore attract universal DM with a force density proportional to gg'^2. We may thus *again expect the density of DM* attracted into a place in a lower-scale structure (where the locally generated field is g) to be Qg, i.e., proportional to the attraction force, or to g, with $\rho_D(0)=0$. So we consider Q to be proportional to g'^2. *Introducing the proportionality factor λ, we obtain*

$$\rho_D = \lambda g g'^2 \equiv Qg. \qquad (2).$$

The important Ansatz $\rho_D = Qg \equiv \lambda g g'^2$ as the DM key for distribution and accretion of universal DM into lower-scale structures was thus obtained in two ways, Eqs. (1) and (2). The second way is physical and less rigorous, but shows that $\rho_D(0)=0$. We use both Q and $\lambda g'^2$ as equivalent, interchangeable symbols. We shall see below that this simple Ansatz $\rho_D = Qg$, *our universal DM key,* is applicable with excellent results in lower-scale structures.

We now apply our DM key to find the accretion of universal DM onto the gravitational field generated by a galaxy in its system of reference. The rotation speed v of a star of mass m, predicted by Newtonian physics in a galaxy with a total baryonic mass M, concentrated uniformly at r<a in the centre, as we assume here, is given for r>a by

$$mv^2/r = GmM/r^2; \quad v = (GM/r)^{1/2}. \qquad (3)$$

In Fig. 1 below, v should follow the dotted lower curve A, but is observed to follow the upper red curve B, showing an almost constant value of v for r>a.

We expect DM generated at the level of the universe to be attracted by the locally generated classical gravitational field $g=GM/r^2$. The DM is attracted proportional to gg'^2 and added to the existing baryonic matter concentration according to our key $\rho_D = Qg = GMQ/r^2$ (for r>a), the key introduced in Eq. (1). For r<a we obtain $\rho_D = Qg = GM(r^3/a^3)Q/r^2$. The total mass $M_<$, found up to the distance r>a from the galactic centre, is then

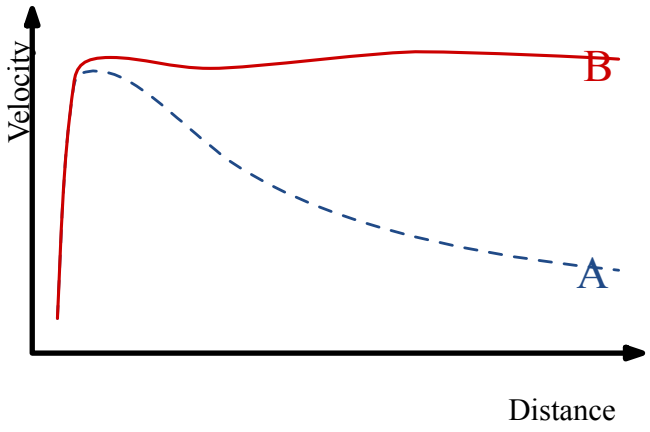

Fig. 1: After reaching a maximum close to the Newtonian prediction A, the speed of stars follows the red curve B and remains roughly constant with distance r. *By Phil Hibbs - his work in Inkscape 0.42, Public Domain, https://commons.wikimedia.org/w/index.php?curid=365013.*

$M_<=M+GQM[\int_0^a r^3/a^3+\int_a^r](4\pi/r^2)r^2dr$

$=M+GQM\int_0^a(4\pi r^3/a^3)dr+4\pi GQM(r-a)$

$=M -3\pi QGMa +4\pi QGMr.$ (4)

Thus, for r>a, the concentration of DM decreases like $1/r^2$, *but the cumulative DM increases proportional to r, creating an extended DM halo of the galaxy. (hose unfamiliar with calculus and integrals should neglect the first term in rectangular brackets and consider that the cumulative DM,* $M_<$. *is the sum of increments dr, thus increases without limit, with r, like r-a). The gravitational field resulting from the Gauss theorem is thus*

$g=M_<G/r^2=MG[1-3\pi QGa]/r^2+4\pi QGM/r.$ (4').

For r≤a we obtain, with a DM concentration $\rho=QGMr/a^3$,

$M_<=Mr^3/a^3+4\pi GQM\int_0^r r^3/a^3dr$

$=Mr^3/a^3+\pi GQMr^4/a^3,$ (5)

showing the complete absence of any DM cusp in the galactic centre.

The rotation speed of stars in galaxies is then, for r>a

$v^2=G(M_<)/r=GM[(1+\pi GQa)/r+4\pi GQ(1-a/r)]=GM[(1-3\pi GQa)/r+4\pi QG].$ (6)

For the first time, this now reveals a constant v^2 of $4\pi QMG^2=M\cdot7.76\cdot10^{-29}cm^2/gs^2$ even for arbitrarily large distances r, as indicated by curve B in Fig.1. This is also shown in Eq. (25) M of the Methods. We expect this constant value of v to continue far beyond the distance up to which the Doppler effect in the 21 cm line of the rotating Hydrogen ring of the galaxy allows us to measure it (a few 10^5 ly). The rotation with constant v should not stop before the gravitational field changes sign, pointing to a neighbouring galaxy (about $1.25\cdot10^6$ ly for the Milky Way).

This was recently verified on both early and late types of isolated galaxies, from the observed weak gravitational lensing, by Mistel et. al. Their results showed no decrease in circular velocity up to distances of 1Mpc = 3,261,564 ly, well beyond the predictions of all existing DM models.

These new measurements clearly verify and validate our GR based theory, eliminating all hypothetical DM models that have been proposed.

For r≤a, we obtain from Eq. (5)

$v^2=G(M_<)/r=GM[r^2/a^3+G\pi Qr^3/a^3],$ (7)

consistent with the absence of a central DM cusp.

These results are valid for any reasonable values of $Q\equiv\lambda g'^2$. ***Eqs.* (6) *and* (7) *exactly reproduce the features of curve B in Fig. 1,* provided $3\pi GQa\approx1$.** In Sec. 3 of the M we apply our DM key to the derivation of the Tully-Fisher relation. Then we use the experimentally well-known constant present in this relation to determine the value of the proportionality factor $Q\equiv\lambda g'^2$. The value of $Q\equiv\lambda g'^2=1.39\cdot10^{-16}gs^2/cm^4$, obtained in Sec. 3 M, yields $3\pi GQa=1.083$. This is slightly larger than 1 and for r>*a* yields a very small, positive derivative of v^2, rapidly decreasing to zero, as seen in curve B, Fig. 1.

By iteration, this effective series expansion in powers of G converges, because the dimensionless GQr is small, as long as $r<10^6$ ly.

Using our DM key we have also calculated the DM in the solar system and found that it yields a negligible correction on the order of 10^{-9} to the mass of the sun at the orbit of the earth. Calculation also showed a solar mass correction of 10^{-4} at a distance of 2 light years (ly) from the sun, beyond the small DM halo of the sun, almost half the distance to the nearest star Proxima Centauri. We also considered qualitatively the DM effects in the Bullet cluster, clusters, and super-clusters of galaxies in general. Finally, we elucidated how DM is dominant at higher scales of the universe and becomes its own source, appearing on both sides of EE. This self-interaction of the gravitational field and of DM leads to their own clumping tendencies and has led, early in the evolution of the universe, to the formation of the cosmic web. The latter is a 3-dimensional system of DM branches loosely filling the universe with a web-like pattern, discovered in the last few decades; it is decorated with galaxies, their clusters and superclusters, with very large voids between branches. The idea of self-interaction of the gravitational field, seen in interactions between gravitons, was also connected earlier with the problem of DM.

3. Derivation of the Dark Energy

We start from the realization that the OU appears to be flat at the largest scale, with a known curvature parameter $\Omega_K=0.0007(19)$, very close to zero[2], as mentioned above. We thus consider a "bumpy observable universe," flat on the larger scale. In this universe, the "dark energy," described by the cosmological constant Λ in EE, entails a mass density of $\rho_\Lambda=5.96\ 10^{-30}$ g/cm^3 that corresponds in EE to the energy density estimated from observations for a flat universe. It compensates for the average

curving effect of matter and field, i.e. of OM and DM. *We derive this dark energy density and its corresponding mass density ρ_Λ here with the help of our DM key, using an equivalent, homogeneous OU, i.e, neglecting small-scale inhomogeneities.*

As an analogy, let's compare the universe to a 2-D square lattice of identical, well-separated, galactic disks of radius L_g, on an infinite plane. Consider the average compensation of the curvature effects in each of the units or "tiles," representing a galaxy, including the appropriate average empty space surrounding it.

In three dimensions this would correspond to a "face-centred cubic" (FCC, close-packed) lattice of galaxies, each of baryonic mass M_g, and radius L_g, with a large surrounding appropriate empty space of size $L>>L_g$, yielding a total volume of $4\pi L^3/3$ per galaxy. This surrounding appropriate space accounts for the many large voids present between the branches of the cosmic web. The actual galaxy size L_g is much smaller than the lattice constant $1.81 \cdot L$ of the primitive (rhombohedral, parallelepipedal) cell of this distorted FCC lattice. The FCC conventional *cubic* unit cell has an edge of 2.56L and contains 4 primitive cells, each with one lattice point.

We can display this compensation of opposing effects on the geometric curvature of the universe in terms of the masses (or equivalent energies) that cause them. *On one hand there is* the total average galactic mass, baryonic (OM) and DM, *on the other is* the dark energy $\rho_\Lambda \cdot L^3$. Here $L^3>L_g{}^3$ is the total average volume, including the appropriate portion of voids, corresponding to a galaxy.

The first two terms in the last form of Eq. (4) almost cancel each other and quickly decrease in importance as r increases, if compared to the last term. To describe this curvature balance at the level of individual galaxies, we retain only the last term in Eq. (4) with $r=L_g$, which is the average radius of a galaxy, including its effective DM halo. The curvature, caused by this term in EE (1) M, is compensated by Λ, i.e. by the curvature effect of the dark-energy-equivalent mass concentration ρ_Λ multiplied by the volume $4\pi L^3/3$. We will display this local compensation as a compensation of the corresponding masses:

$$4\pi G Q M_g L_g - \rho_\Lambda 4\pi L^3/3 = 0. \qquad (8)$$

(An equivalent geometric interpretation and derivation of this compensation is suggested in Supplement 1).
Eq. (8) yields

$$\rho_\Lambda = 3GQM_gL_g/L^3. \qquad (9)$$

With the help of our universal DM key, its coefficient $Q = 1.39 \cdot 10^{-16} gs^2/cm^4$, and with $G=6.67 \cdot 10^{-8}\ cm^3/gs^2$, we obtain

$$\rho_\Lambda = 3 \cdot 6.67 \cdot 10^{-8} \cdot 1.39 \cdot 10^{-16} cm^{-1} M_g L_g/L^3 = 2.78 \cdot 10^{-23} cm^{-1} \cdot M_g L_g/L^3. \qquad (9')$$

We introduce the average baryonic mass of a galaxy in the OU, also used in Eq. (16) M below,

$$M_g = 1.4 \cdot 10^{43} g. \qquad (10)$$

As previously mentioned, the value of the outer radius L takes into account the many large voids present between the branches of the cosmic web. This web may contain about 2.10^{12} galaxies. L is obtained by scaling down the radius R_u of the OU by a factor of $(2 \cdot 10^{12})^{1/3}$, yielding

$$L = (2 \cdot 10^{12})^{-1/3} \cdot 4 \cdot 10^{28}\ cm = 3.175 \cdot 10^{24}\ cm$$
$$= 3.34 \cdot 10^6\ ly;\ L^3 = \mathbf{3.2 \cdot 10^{73}\ cm^3} \qquad (11)$$

Considering [here and in Eqs. (15)-(16) M] an average effective radius $L_g=D$ of the galactic disk, including the main part of the rotating hydrogen belt and DM halo, $L_g = 5 \cdot 10^5$ ly $= 4.75 \cdot 10^{23}$ cm, we obtain from Eq. (9)

$$\mathbf{\rho_\Lambda} = 2.78 \cdot 10^{-23} \cdot 1.4 \cdot 10^{43} \cdot 4.75 \cdot 10^{23} / 3.2 \cdot 10^{73} g/cm^3 = \underline{\mathbf{5.8 \cdot 10^{-30}\ g/cm^3}}. \qquad (12)$$

This is in very good agreement with the observed dark energy density value known to be $\rho_\Lambda = 5.96\ 10^{-30}\ g/cm^3$. The error is less than 3%. The corresponding cosmological constant is thus

$$\mathbf{\Lambda = (8\pi G/c^2)\rho_\Lambda} = 8 \cdot 3.14 \cdot 6.67 \cdot 10^{-8} \cdot 5.8 \cdot 10^{-30}\ cm^{-2} / 9 \cdot 10^{20} = \mathbf{1.08 \cdot 10^{-56}\ cm^{-2}}. \qquad (13)$$

This is again very close to the known value of $\Lambda = 1.088 \cdot 10^{-56}\ cm^{-2}$. This is a first simple derivation of the cosmological constant, based in fact on Einstein's equation, on its local Newtonian approximation, and on our "DM key." This is a first solution of the "cosmological constant problem."

These very close results are very remarkable, but also fortuitous in their extreme precision, given the uncertainties in L_g, L and M_g. However, even the order of magnitude of this agreement is very remarkable. It shows how well our "DM key" works to approximate and anticipate the results from the nonlinear EE. Indeed, EE is known to be prohibitively difficult to solve exactly, even more so for all interacting scales of the universe.

Our DM key also describes the large DM effects observed in collisions between clusters of galaxies, as seen in the well-known Bullet cluster and in the recently studied MACS J0018.5+1626 clusters merger. In both cases the DM seems to outrun the intra-cluster hot ionized gas that provides most of the baryonic mass. Our DM key yields a DM distribution centred on the hot gas, but which is zero in the centre of the mass distribution and large only outside the remote peripheric regions. Indeed, the DM

density ρ_D=Qg is expected to peak farther, out of the less massive front-and-end tip of the mass distribution, the tip where the stars that went through the collision centre are located. However, the maximum of the cumulative DM $M_<$ is reached far outside the front-end tip P, or outside the rim of the mass distribution, and far behind the end tip P', (on the other side, on the side of the stars of the smaller of the colliding clusters that went through the collision centre, as observed). The DM density ρ_D decreases with the distance r from the front point P (or end point P') slightly faster than $1/r^2$. *However, the cumulative $M_<$ increases strongly, almost proportional to r, peaking very far away out from the end points, as observed. This is similar to the seemingly unlimited increase of M< in halos of isolated galaxies, Eq. (4). For galaxy clusters the cumulative DM predicted by our key also increases proportionally to the distance r from P or r' from P', but slightly more slowly*, with a density slightly below $1/r^2$, because the OM distribution is very elongated, and not spherically symmetric. This creates the false impression of the DM outrunning OM at both ends.

To also cover the even higher and highest scales S, with S>>L', of the OU above the "reference length" of about $L'\approx10^9$ ly, for the density of DM we could write

$$\rho_D=gQ(L'/S)Log(1+S/L'). \quad (14)$$

This extended DM key, applicable for any scale (or characteristic length S of the structure considered, including S<L'), can be used for possible "hyper-clusters of superclusters," if found, and for ***all scales*** $\mathbf{0<S\le R_u}$ (where the additional factors in Eq. (14) are reduced to unity for S<<L'). From Eq. (14) we see that it lowers the values of $Q=\lambda g'^2$ and ρ_D from the lower-scales amount in Eq. (16) M for all higher scales S>L'. Eventually, the reference length L' could be fine-tuned, so the DM density (14) would add up to the total DM (about five time the OM) in the OU. Eq. (14) needs to be tested and verified (see Supplement1).

Indeed, an important part of the universal DM will be accreted on the inter-galactic ridges of gravitationally self-interacting aggregate of "DM plus inter-galactic gas and dust," generating the branches of the cosmic web. Furthermore, the DM amounts defined in the rest frames of lower-scale structures may not add up to the universal DM (see Supplement 1).

We gladly acknowledge useful discussions with C. D. H. Frederick, T. F. George and E. L. Gibb.

METHODS

1. Dark Matter in the Observable Universe

This section of Methods presents an alternative derivation, the initial derivation, based on EE and an electromagnetic analogy. In the first two sections here, it yields the rather simple, transparent and intuitive results for the percentage of DM in the OU.

Einstein's equations contain the metric tensor $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, defining the gravitational potential and field, *on both sides,* both from the resulting $g_{\mu\nu}$, and on the r. h. s. as part of its source, being present in $t_{\mu\nu}$, the gravitational field part of $T_{\mu\nu}$:

$$R_{\mu\nu}-g_{\mu\nu}\mathscr{R}/2-\Lambda g_{\mu\nu}=(8\pi G/c^4)T_{\mu\nu}. \quad (1)$$

Here $R_{\mu\nu}$ is known as the Ricci tensor, obtained by contraction from the curvature tensor $R^{\lambda}_{\mu\nu\sigma}$, $\mathscr{R}$ is the curvature scalar obtained from it by further contraction, Λ the cosmological constant that describes the level of DE, $T_{\mu\nu}$ is the total energy-momentum-stress tensor, G Newton's constant of universal gravitation and c the speed of light. The field of the deviations $h_{\mu\nu}$, from the Euclidian metric (1, -1, -1, -1), contributes a gravitational field part, $t_{\mu\nu}$, to the total energy-momentum tensor $T_{\mu\nu}$. An expression for $t_{\mu\nu}$, a restricted, or quasi-tensor, up to second order in $h_{\mu\nu}$, is given by Weinberg[4] in Eq. 7.6.14. We used this expression for t_{00} to calculate the total DM in energy units, W_{tot} in the OU analytically,[3] to second order in $h_{\mu\nu}$ only, in spherical symmetry and in cylindrical flow symmetry. We found $W_{tot}=1.2\ Gm^2N^2/R_u$, in spherical symmetry, where R_u is the radius of the OU, and N is the number of galaxies of average mass m in the universe. By dividing W_{tot} with the rest energy of OM, Nmc^2, we obtain the ratio of DM to OM in the OU

$$s''=1.2(4/3)Gm^2N'N/Nmc^2=0.8N'r_s. \quad (2)$$

This result, $0.8N'r_s$, is twice the classical result, derived[3] with quasi-Newtonian theory. Here we introduced $N'=3N/4R_u$, the number of galaxies per unit length in the z-direction at the "equator" of the spherical model of the OU. We denoted by r_s the average Schwarzschild radius $r_s=2Gm/c^2$ of a galaxy, with m its average mass. When the universe is considered as a column moving with a certain hyperbolical angle to the time axis and (eventually virtual) spatial velocity **v** in the z-direction, we show below, that the ratio of DM to OM should indeed be $s''=N'r_s$, as displayed in Eq. (2). Here N' is again the number of galaxies per unit length of the column.

To verify, we note that each galaxy moves along its world line. Consider the OU as the 3 D cross section of a column moving along its world corridor. Consider it also in 3 dimensions as a long column embedded in the rest of the universe, moving along its axis with a velocity vector **v** (eventually virtual), determined by the hyperbolic angle to the time axis.

Starting from an electrodynamic analogy to electric currents in a semiconductor or metallic wire, we show that, in the analog gravitational case, the DM ratio to OM in mass or energy is $N'r_s$, with the above notations. We mention this analogy here, both for historic reasons, and since it makes the notion of DM more accessible to a general readership.

Indeed, (To see this,) the kinetic energy of the ordered motion of electrons with drift velocity v in a wire of radius *a'* carrying the current J=N'ev, with N' electrons per unit length, has two components: the coherent (many-body) magnetic field energy

$\int B^2 d\tau/8\pi$, proportional to N'^2 in cylindrical symmetry, and the (incoherent) mechanical energy $N'mv^2/2$, proportional to N'. Their ratio is

$$s = [\int B^2 d\tau/8\pi]/[N'mv^2/2] = \{\iint [4J^2/8\pi c^2][2\pi r dr/r^2]\}/\{N'mv^2/2\}$$
$$= e^2 N'^2 v^2 [(0.25+\ln(R_0/a'))]/[c^2 N'mv^2/2]$$
$$= 2N'(e^2/mc^2)[0.25+\ln(R_0/a')] \approx 2N'r_e. \quad (3)$$

Here $r_e = e^2/mc^2 = 2.8 \cdot 10^{-13}$ cm is the classical radius of the electron in CGS Gauss non-rationalized units.

The logarithm is explained after Eq. (4) below. By replacing $\mathbf{B}=\mathbf{v}x\mathbf{E}/c$ with a quasi-Newtonian $\mathbf{v}x\mathbf{g}/c$ for the kinetic gravitational field, we obtain[3] the same way as *s for* the electrical case, *in the gravitational case* $s''=N'r_s$. This parameter s" is again *independent of* **v**. *It* is the coherent (many-body) kinetic gravitational field energy of the galaxies, divided by the mechanical $N'mv^2/2$, both per unit length along the world corridor of radius $R_u=a$ of the OU in 3 dimensional space:

$$s'' = s_g = [\iint (\mathbf{v}x\mathbf{g}/c)^2 d\tau/8\pi G]/[N'mv^2/2] = [4Gm^2N'^2v^2/(8\pi c^2 N'mv^2/2)] \iint 2\pi r dr/r^2$$
$$= 2(GN'm/c^2)[0.25 + \ln(R_0/a)] \approx 2N'Gm/c^2 = N'r_s. \quad (4)$$

Here $r_s = 2Gm/c^2$ is again the average Schwarzschild radius of a galaxy. In Eqs. (3) and (4) the field energy *inside* the wire of radius *a'* and *inside* the radius *a* of the column, was included, and displayed explicitly both times in the last rectangular brackets through the term 0.25. The logarithm describes a cutoff of the integrals by imagining an arbitrarily large closed circuit of radius R_0. The expression in rectangular brackets, containing the logarithm, was approximated with the unity in the final result.

Our Ansatz $\mathbf{v}x\mathbf{g}/c$ in Eq. (4) was verified in Sec. 2.4 of an earlier paper[3], based on an approximation of $t_{\mu\nu}$, derived from EE. A discussion of the electromagnetic analogy is given Weinberg[4] pp. 161-171, and by Williams and Inan.

In our analogy, the kinetic energy part from the magnetic field in a semiconductor or metallic wire corresponds to the coherent kinetic gravitational energy part of the universe, that manifests itself as its DM mass times $v^2/2$. Our detailed, laborious, calculations[3] of this DM, both classically and in low-order GR, both in cylindrical flow and in static spherical symmetry, have strengthened our confidence that a rigorous solution of EE would indeed yield *5 times more DM* (as defined by us) *than OM in the OU. It inspired the subsequent successful exact calculation of DM in all lower-scale structures of the universe with our DM key, shown here in the Main Text.*.

For example if all galaxies would be like our Milky Way with $M \approx 10^{45}$ g, we would obtain

$$r_{s(gal)} = 2GM/c^2 = 13.3 \cdot 10^{-8} \cdot 10^{45}/9 \cdot 10^{20}$$
$$= 1.48 \cdot 10^{17} \text{cm}. \quad (5)$$

Thus, in the OU with radius $R=R_u=46.5 \cdot 10^9$ ly $= 4.4 \cdot 10^{28}$ cm, with about $N=2 \cdot 10^{12}$ galaxies, we can approximate the number of galaxies per unit length in cylindrical flow symmetry, by multiplying the average concentration of galaxies $3N/4\pi R_u^3$ with the equatorial area of the OU, πR_u^2:

$$N' = [2 \cdot 10^{12}/1.33\pi R_u^3][\pi R_u^2] = 6 \cdot 10^{12}/4R_u$$
$$= 1.5 \cdot 10^{12}/4.4 \cdot 10^{28} \text{cm} = 3.4 \cdot 10^{-17} \text{ cm}^{-1}. \quad (6)$$

Therefore, we obtain

$$N'r_s = 3.4 \cdot 10^{-17} \text{cm}^{-1} \cdot 1.48 \cdot 10^{17} \text{cm} = \mathbf{5.032.} \quad (7)$$

The universe would thus contain about 5 times more DM as OM, in agreement with the present standard "Λ cold dark matter" (ΛCDM) model, if our Milky Way could be considered an average galaxy. As we see below, it's above average, but DM becomes its own source to considerable extent from EE, and that seems to make up for it. This exact result may thus seem to be fortuitous, but the order of magnitude is important.

This interdisciplinary paper and the electromagnetic analogy used above were inspired by (the gravidynamic form of) our *quantum theory* of the ubiquitous *fundamental 1/f noise (Q1/fN)*, practically important in its QED form, in electrical engineering[3] and in most of high-technology.' Anticipating the modern theory of decoherence by decades, it used the "Einselected states," long before they were discovered. Due to this anticipation, Q1/fN was misunderstood by some vocal theorists 10 years after its successful introduction, and was discredited erroneously. However, Q1/fN explains the fundamental, universal, 1/f noise and is expressed by simple universal engineering formulas for the new, "physical cross sections and process rates" of quantum mechanics, *and coherently, also for any (electrical or neutral-matter) current*. This is useful for the current US CHIPS program. Its "Einselected states[11]," or "pointer states" are more stable to the dephasing interactions with the environment. *Indeed, Eqs. (3) M and (4) M also yielded the ratio between coherent and conventional quantum 1/f effects in the gravidynamic quantum theory of 1/f noise*. Like the later-developed, well-known theory of decoherence (see, e.g.,[11]), which it anticipated since 1975, this quantum 1/f theory represents a new basic aspect of quantum mechanics, bridging the "quantum divide" to our macro-world.

2. Dark Matter Generated in Lower-Scale Structures

In lower level structures the own, classically generated, Newtonian local gravitational field corresponds to a negligible energy or mass, compared with the OM. Indeed, even for a whole disk-shaped spiral galaxy, with 10^{11} stars moving *along the axis of the galaxy* with disk-thickness 10^4 ly=$9.5\cdot10^{21}$ cm, we obtain

$$N' =10^{11}/10^4\text{ly} =10^{11}/9.5\cdot10^{21}\text{ cm}$$
$$=1.05\cdot10^{-11}\text{ cm}^{-1}. \quad (8)$$

Considering the mass of a star $2\cdot10^{33}$g, like the mass of the sun, with $r_s=2GM/c^2=3$km,
we obtain

$$N'r_s=1.05\cdot10^{-11}\text{ cm}^{-1}\cdot3\cdot10^5\text{cm}=3.15\cdot10^{-6}. \quad (9)$$

This is a very small fraction of DM that is generated at the level of a galaxy, compared to OM. If we consider (real or virtual) motion of the galaxy along a diameter of the disk, we obtain an about 20 times smaller result. *In conclusion, the amount of DM generated in galaxies and in the lower levels of the universe is very small and can't explain the observed DM effects. Historically, this was a main reason for introducing DM.*

If applied to the lower level structures of the universe, and not to the whole OU, $N'r_s$ tends to over-estimate somewhat the ratio DM/OM; this does not affect our conclusion.

The over-estimation occurs because in the case of a lower-scale structure, e.g., galaxy, we do not have an infinitely long homogeneous column mowing along its axis, but rather a finite flow of matter, e.g., stars that has a smaller kinetic gravitational field and field energy (like the magnetic field of a single moving electron), than an infinitely long column with the same N'. The whole OU, on the other hand, can be assumed to move along a path in 3D with a certain (eventually virtual) velocity, along with parts of the surrounding universe preceding it and following it in a larger extrapolated stream.

3. The Tully-Fisher Relations

These relations state that *the baryonic mass M of a galaxy is proportional to about the fourth power of the peripheral speed* v *of the OM in it:*

$$M/M=58(v/1\text{km}\cdot\text{s}^{-1})^{3.97}, \quad (10)$$

were M is the mass of the sun. In Eq. (4') of the Main Text (MT) the first term in rectangular brackets is proportional to 1/r and decreases to zero already for r<<D. We can neglect it at larger r values and in general. Thus, by substituting for r in Eq. (4') MT the total radial extension L_g=D of the galaxy including the halo, we obtain with the Gauss Theorem $\oint\mathbf{g}ds=4\pi GM_t$, a good approximation of the total (OM + DM) mass M_t of the galaxy

$$M_t\approx4\pi G\lambda QMD. \quad (11)$$

Its total gravitational energy in the NA is thus

$$G(M_t)^2/2D=\gamma Mc^2. \quad (12)$$

Here we compared this gravitational energy, with the rest energy Mc^2 of OM in the galaxy and introduced γ as their ratio. The maximal rotational speed, mainly of hydrogen, at the periphery of the galaxy is then given by

$$v^2=GM_t/D=[2\gamma GMc^2/D]^{1/2}. \quad (13)$$

In the last form we have substituted M_t from Eq. (12). From Eq. (13) *we derive the Tully-Fisher relations in the form*

$$\boldsymbol{v^4=2\gamma GMc^2/D} \quad (14)$$

in which the exponent 3.97 is replaced by 4. ***This is the first general derivation of the Tully-Fisher relations,*** *from first GR (and local NA) principles to our knowledge.*

Eq. (14) allows for a determination of the coefficient γ by comparing with Eqs. (10) and (14), in CGS units, with $D=5\cdot10^5$ ly $=4.7\cdot10^{23}$cm, introduced in Eq. (12) MT,

$$2\gamma =D(1\text{km/s})^4/58Gc^2M$$
$$=4.7\cdot10^{23}\cdot10^{20}/58\cdot6.7\cdot10^{-8}\cdot9\cdot10^{20} \quad \cdot2\cdot10^{33} =6.8\cdot10^{-6}. \quad (15)$$

This yields $\gamma=3.4\cdot10^{-6}$ for a galaxy.

Eq. (15) allows in turn, to derive the quantity $Q\equiv\lambda g'^2$ in Eq. (1) MT, which describes the accretion of DM generated at the level of the observable universe into lower-scale structures. Indeed, by eliminating M_t, from Eqs. (11) and (12), we obtain from Eq. (10), taking, like in Eq. (10) MT the baryonic mass of an average galaxy to be $M=M_g=1.4\cdot10^{43}$g,

$$Q =(c/4\pi)(\gamma/MDG^3)^{1/2} =(3\cdot10^{10}/4\pi)[6.7\cdot10^{-6} /1.4\cdot10^{43}\cdot4.7\cdot10^{23}\cdot(6.7)^3\cdot10^{-24}]^{1/2}\ {}^{16}\text{gs}^2/\text{cm}^4$$
$$\equiv Q=(2.39\cdot10^9)[3.4\cdot10^{-51}]^{1/2}$$
$$=1.39\cdot10^{-16}\text{gs}^2/\text{cm}^4. \quad (16)$$

This defines the coefficient of the universal DM accretion key, determining the amount of DM found in galaxies, their clusters, and in all lower-scale structures, like, e.g., planetary systems, binary stars, or planets. As shown in Eqs. (9)-(12) MT, this has allowed

also for the first calculation of DE and of the cosmological constant. From Eqs. (15) and (16) we see that $Q \equiv \lambda g'^2$ does not depend on the value of D.

Furthermore, Eq. (16) fixes the relative value of the DM terms with λ or Q in Eqs. (4)-(7) MT as was claimed above, right after Eq. (7) MT. *At r=a*, the DM terms in rectangular brackets in Eq. (6) MT together yield one third of the DM term in round brackets. This is to be compared with *1/a*, which comes from the first term in round brackets, the baryonic contribution. Approximating *a* in Eq. (6) MT with the size of the main central baryonic mass of the Milky Way, of $a=1.3\cdot10^4$ly (about the size of the central bar of the Milky way), we obtain with Eq. (16)

$3\pi QGa=9.42\cdot1.39\cdot10^{-16}6.7\cdot10^{-8}1.3\cdot9.5\cdot10^{21} \quad =1.083.$ (17)

This is indeed slightly larger than 1, as we mentioned above. Therefore, we obtain from Eq. (6) MT for r≥a:

$$v^2=GM[(1-1.083)/r+1.17\cdot10^{-22}cm^{-1}]$$

$$=M\{-5.56\cdot10^{-9}/r+7.84\cdot10^{-30}cm^{-1}\}cm^3/gs^2, \quad (18)$$

which yields a small, fast decreasing, positive derivative $(d/dr)(v^2)$ at *r>a*, in very good agreement with the observed curve B in Fig. (1). The slight undulation and later small bulging on curve B can be related to the presence of stars at distances larger than *a*.

For r<a we obtain from Eq. (7)

$$v^2=GM[r^2/a^3+2.92\cdot10^{-23}cm^{-1}r^3/a^3]. \quad (18')$$

This result ignores at r<<a the presence of a black hole at r=0 in most galaxies.

The consistence of the exact results obtained with our DM key verifies our physical approximation of unknown exact GR results, expected from EE. Moreover, we proved here that our theory does not contain any free, adjustable, arbitrary, parameters or "fudge factors."

The exact calculation of DM density in any point, of DM content in any volume on the earth, could also allow direct interferometric verification with recently found, ultra-sensitive devices.

4. Application of our DM Key to MOND.

The "modified Newtonian dynamics" (MOND) claims" that Newton's law of universal gravitation, and implicitly GR, break down at very low accelerations below 10^{-9} m/s^2 (corresponding to very large distances from a large central mass). According to the very successful MOND, the $1/r^2$ decrease of gravitational acceleration at large distances r stops and needs to be replaced by a constant a_0, or by a geometric mean between the Newtonian gravitational field g_N and a_o. *We show here that MOND and its partial successes can be understood on the basis of our DM key*. We are thus relying on EE and its NA, rather than demonstrating their breakdown.

According to the MOND algorithm, all very small Newtonian gravitational accelerations $g_N<10^{-8}$ cm/s^2 need to be replaced by g from

$$g^2=g_Na_o. \quad (19)$$

In general, g_N is replaced by *MOND*[17,18,19] through $g=\nu(g_N/a_o)g_N$, where

$$\nu(x)\to1 \text{ for } x>1 \text{ and } \nu(x)\to x^{-1/2} \text{ for } x<1. \quad (20)$$

We show here that the approximate solution of the EE and its NA expressed by our DM key validates the phenomenological MOND model to large extent. It explains the nature of DM and derives an analytical formula that defines a lowest acceleration a_o for any galaxy, cluster, or other cosmic system of masses.

As we see from Eq. (6) MT, for a point mass M (or "galaxy" with central mass geometrical size *a=0)*, at a distance *r*, the centripetal acceleration g of a star in a galaxy is obtained with our key from

$$v^2/r=(GM/r)\{1/r+4\pi GQ\}=g$$

$$=(GM/r)\{1/r+1.164\cdot10^{-20}cm^{-1}\}. \quad (21)$$

By squaring Eq (21), we can write

$$g^2=v^4/r^2=(GM/r^2)\{GM/r^2+(G^2M/r)8\pi Q+MG^3(4\pi Q)^2\}. \quad (22)$$

The last term in curly brackets, surviving at all larger distances r, yields *our remaining minimal acceleration* a'_o. It does not decrease with r, and exactly corresponds to the definition in Eq. (19), with $g_N=GM/r^2$:

$$a'_o=MG^3(4\pi Q)^2 \mathbf{=9\cdot10^{-52}\ (cm/s^2g)M}$$

$$=1.26\cdot10^{-8}cm/s^2=\mathbf{1.26\cdot10^{-10}m/s^2.} \quad (23)$$

The last two forms of Eq. (23) are again for a galactic central mass of $M_g=1.4.10^{43}$g. Our result for a'_o is in general agreement with the $a_o\approx10^{-10}$ m/s^2 introduced by Milgrom and others for galaxies and clusters thereof. However, we notice that our smallest acceleration a'_o is proportional to the central mass M. *Our universal constant* defined by Eq. (23) is

$$a'_o/M=G^3(4\pi Q)^2=9\cdot10^{-52} \text{ cm/s}^2\text{g}. \quad (24)$$

Another universal constant we introduce is the asymptotic squared velocity per unit central mass $v_o^2/M=g\cdot r/M$

$$v_o^2/M=4\pi QG^2=\mathbf{7.76\cdot10^{-29}cm^2/gs^2.} \quad (25)$$

This is also a universal constant that we introduce here.

We note that $4\pi\lambda g'^2G \equiv 4\pi QG = 1.164 \cdot 10^{-20}$ cm^{-1} $= 1/r_o$, *where r_o=9000 ly is the universal distance at which the centripetal acceleration caused by DM gets to equal the one caused by OM, in the special case a=0, with M concentrated at r=0.*

It is well known that MOND runs into difficulties at scales larger and lower than galactic. It does not work at scales below 1 light year, and struggles in clusters of galaxies.[21] Our derivation shows that no modification of Newton's or EE is needed in order to eliminate the need of hidden DM particle approaches in ΛCDM at any scale.

This concludes our derivation from EE, and the generalization for all scales, of the MOND phenomenological approximation.

The application of our theory to binary stars is shown in Supplement 1.

See Supplement 1: "The Nature of Dark Energy and Dark Matter, and Application to Wide Binaries" URL to be determined by Editors.

References

1. M. Tanabashi et. al. Astrophysical Constants and Parameters. Physical Review D. Particle Data Group. 98, 030001 (2019); doi:10.1103/PhysRevD.98.030001.
2. Planck 2018 results. VI. Cosmological parameters. arXiv>astro-ph >arXiv:1807 .06209.
3. P. H. Handel and K. E. Splett, Calculation of Dark Matter as a feature of Space-Time, Springer Nature "Foundations of Physics," 53 (5) 2023, DOI 10.1007/s10701-023-00705-x.
4. S. Weinberg, Gravitation and Cosmology. Wiley and Sons, 1972.
5. T. Mistele, S. McGaugh, F. Lelli, J. Scombert and P. Li, Indefinitely Flat Circular Velocities and the Baryonic Tully-Fisher Relation from Weak Lensing, arXiv:2406.09685v1 [astro-ph.GA] 14 Jun 2024
6. A. Deur, Implicatio It does not work at scales below 1 light year, and struggles in clusters of galaxies.21ns of Graviton-Graviton Interactions on Dark Matter. Physics Lett. B 676, 21-24 (2009).
7. E. M. Silich et al, ICM-SHOX. I. Metho-dology Overview and Discovery of a Gas–Dark Matter Velocity Decoupling in the MACS J0018.5 +1626 Merger, Ap. J. 968, 74 (2024) DOI 10.3847/1538-4357/ad3fb5e.
8. L. L. Williams and N. Inan, Maxwellian Mirages in General Relativity. New J. Phys. 23, 053019 (2021).
9. A. van der Ziel, Unified Presentation of 1/f Noise in Electronic Devices; Fundamental 1/f Noise Sources, Proc. of the IEEE, 76, 233-258 (1988).
10. P. H. Handel and A. G. Tournier, Nanoscale Engineering for Reducing Phase Noise in Electronic Devices. Proc. of the IEEE 93, 1784-1814 (2005).
11. W.H. Zurek: Decoherence, Einselection and the Existential Interpretation (the rough guide), Phil. Trans. R. Soc. Lond. A (1998) 356, pp. 1793-1821.
12. P.H. Handel: "Coherent and Conventional Gravidynamic Quantum 1/f Effects," Proc. Intl. Conf. on Noise and Fluctuations (ICNF 2019 Neuchatel) Eds. Ch. Enz and F. Pascal, Imprimerie EPFL Lausanne, pp. 140-143.
13. T. Folger, Scient. American July 2018, p. 30-35
14. J. Trachternach, J. et al. The baryonic Tully-Fisher relation and its implication for dark matter halos. Astronomy and Astrophysics, 505, 577-587 (2009).
15. Yiping Wang et. al., Robust Quantum Control via Multipath Interference for Thousandfold Phase Amplification in a Resonant Atom Interferometer. Phys. Rev. Lett. 133, 243403 – Published 11/12/24 DOI https://doi.org/10.1103/PhysRevLett.133.243403
16. M. A. Milgrom, Modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. Astrophysical Journal 270, 365–370 (1983)
17. M. Milgrom, A modification of the Newtonian dynamics - Implications for galaxies. Astrophys. J. 270, 371–383 (1983)
18. M. Milgrom, A modification of the Newtonian dynamics - Implications for galaxy systems. Astrophys. J. 270, 384 (1983).
19. J. Beckenstein and M. Milgrom, Does the missing mass problem signal the breakdown of Newtonian gravity? Astrophys. J. 286, 7-14 (1984).
20. Milgrom, M., Models of modified-inertia formulation of MOND, Phys. Rev. D, 106, 064060 (2022).
21. S. Mazurenko, I. Banik and P. Kroupa, A simultaneous solution to the Hubble tension and observed bulk flow within 250 h−1 Mpc, MNRAS 527, 4388–4396 (2024) https://doi.org/10.1093/mnras/stad3357.