Quantum Theory of 1/f Frequency Fluctuations Part 2

Quantum 1/f Optimization of Quartz Resonators, Electronics, MEMS, Clocks, Piezotransducers, and Resonant Sensors

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Abstract:
The present poster gives examples of Quantum 1/f (Q1/f) optimization of sensors and systems. It investigates the application of the quantum theory of 1/f noise to the optimization of quartz and Si MEMS sensors for ultra-low 1/f noise and phase noise close to carrier. This multi-disciplinary poster is transformative, because it allows an order of magnitude increase of sensitivity, detectivity, and stability, based on this new fundamental aspect of quantum mechanics we have discovered. It provides simple engineering formulas that allowed us to give optimization rules for both resonant and non-resonant sensors. This includes quartz microbalances, bio-chemical sensors based on BAW and SAW resonators of quartz or other piezoelectric materials, MEMS/NEMS resonant and non-resonant sensors, FET and HFET-based detectors, MQW and QWIPs photodetectors, infrared junction and MIS detectors, quantum dot detectors, etc, Due to the novelty of the field, we focus here both on the basics and on practical examples of optimization for improved radar and sonar range resolution, and for optimizing the piezoelectric transducers for minimal quantum 1/f size fluctuations. Practical examples of optimization will be shown, also for quartz and for Si MEMS resonators, as well as mixers, local oscillators, and clocks. This will induce an across the board order of magnitude improvement revolution in hi-tech hardware across the DOD branches, including drastic casualty reduction from IEDs for expeditionary corps.

The Quantum 1/f Effect, Coherent and Conventional, is a **new aspect of Quantum Mechanics** that will revolutionize Military and Civilian Technology, Nanotechnology devices and systems through **Q1/f Optimization for ULTRA-LOW NOISE &** PHASE NOISE. **BASED ON DECOHERENCE!**

1/f Noise and PHASE NOISE close to carrier limits performance in most Hi-Tech Devices & Systems. Unlike all other forms of noise, YOU CAN NOT AVERAGE IT OUT. For the first time now, we understand it completely, WE CAN CONTROL IT, and REDUCE IT VERY MUCH BY QUANTUM 1/F OPTIMIZATION OF DEVICES AND SYSTEMS.

Quantum 1/f Fluctuations Basic Formulas: Power Spectral Density S(f) of current j; $\alpha=2\pi e^2/hc=1/137$;

$$S_{\delta\sigma/\sigma}(f) = 4\alpha(\Delta v/c)^2/3\pi fN \quad \text{Conv}$$

$$S_{\delta j/j}(f) = 2\alpha/\pi fN = 4.6\ 10^{-3}/fN \quad \text{Coher}$$

$$S_{\frac{dR}{R}} \quad \frac{1}{1 \quad s} S^{conv}_{\frac{dR}{R}}(f) \quad \frac{s}{1 \quad s} S^{coh}_{\frac{dR}{R}}(f)$$

where $\Delta \mathbf{v}$ is the change in velocity vector of the charge carriers in the scattering process, N is the number of particles that defines the scattered current **j**, $s=2N'r_o$, N' is the number of carriers per unit length, $r_o=e^2/mc^2$ is the classical radius of the electron, R is the resistance or the current or the voltage. Q1/fE ARE BASED ON DECOHERENCE

Conventional Q1/fE in mobility. Matthiessen

$1/\mu=\Sigma 1/\mu_j$; $\delta\mu/\mu^2=\Sigma\mu_j/\mu_j^2$

$S_{\delta\mu/\mu}(f)=\Sigma(\mu/\mu_j)^2 S_{\delta\mu/\mu_j}(f)$.

Impurity scattering: $S_{\delta\mu/\mu}(f)=S_{\delta\sigma/\sigma}(f)$.

$S_{\delta\mu/\mu}(f)_i=S_{\delta\sigma/\sigma}(f)_i=(4\alpha/3\pi fN)<[6k_BT/m^*c^2]>$
$\approx(10^{-9}/fN)(Tm_o/4m^*100K)$.

Normal acoustic phonon scattering.

$S_{\delta\mu/\mu}=(4\alpha/\pi fN)<(\Delta\mathbf{k}/m^*c)^2>$
$=(3.10\text{-}3/fN)<(\Delta\mathbf{q}/m^*c)^2>$
$=S_{\delta\mu/\mu}(f)=(3.75.10\text{-}8/fN)(m_e/m^*)^2$

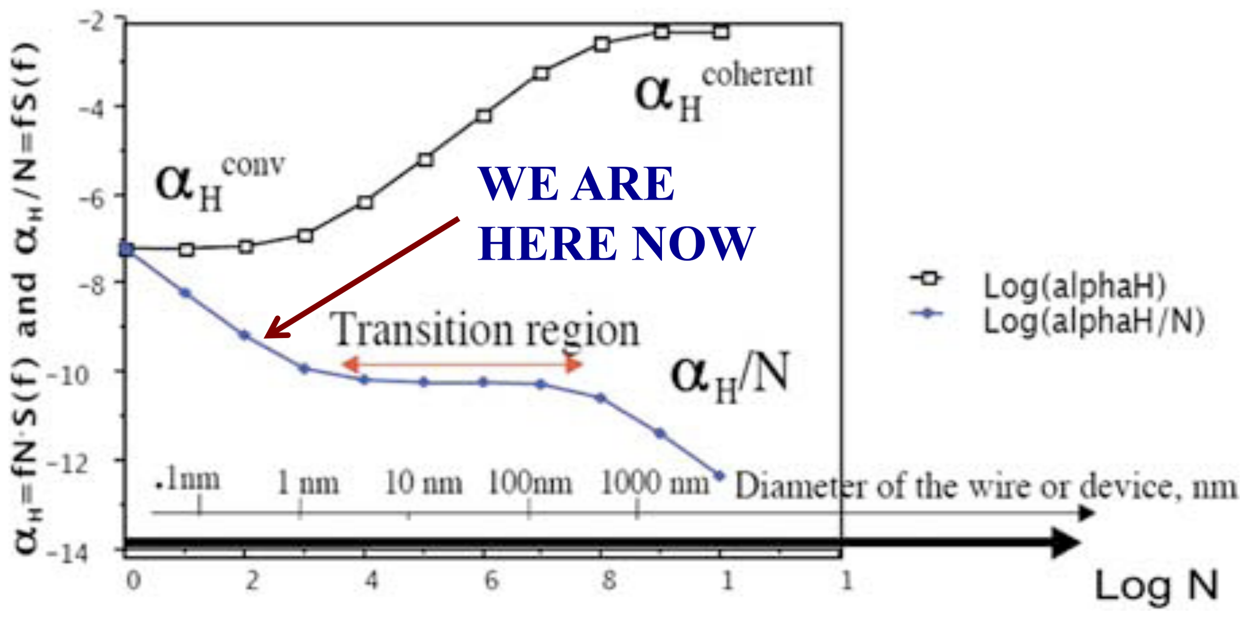

Fig. 1: The Q1/f parameter α_H and spectrum of fractional mobility fluctuations $\mathbf{fS_{\delta\mu/\mu}=\alpha_H/N}$ shown as a function of the number N of carriers in the sample. The transition region, in which we observe the proximity effect, is marked by a red arrow. The wire diameter scale gets shifted to the right (or left) proportionally when the concentration of carriers increases (or decreases) from the assumed value of 10^{17} cm^{-3}. We took the sample cross section to be (140nm^2), the length 5 mm, and the concentration n = 10^{17}cm^{-3} as an example. The flat portion of the lower curve explains why 1/f noise was less of a problem during the past miniaturization stage. It is now again a problem, as the increase below logN=3 shows.

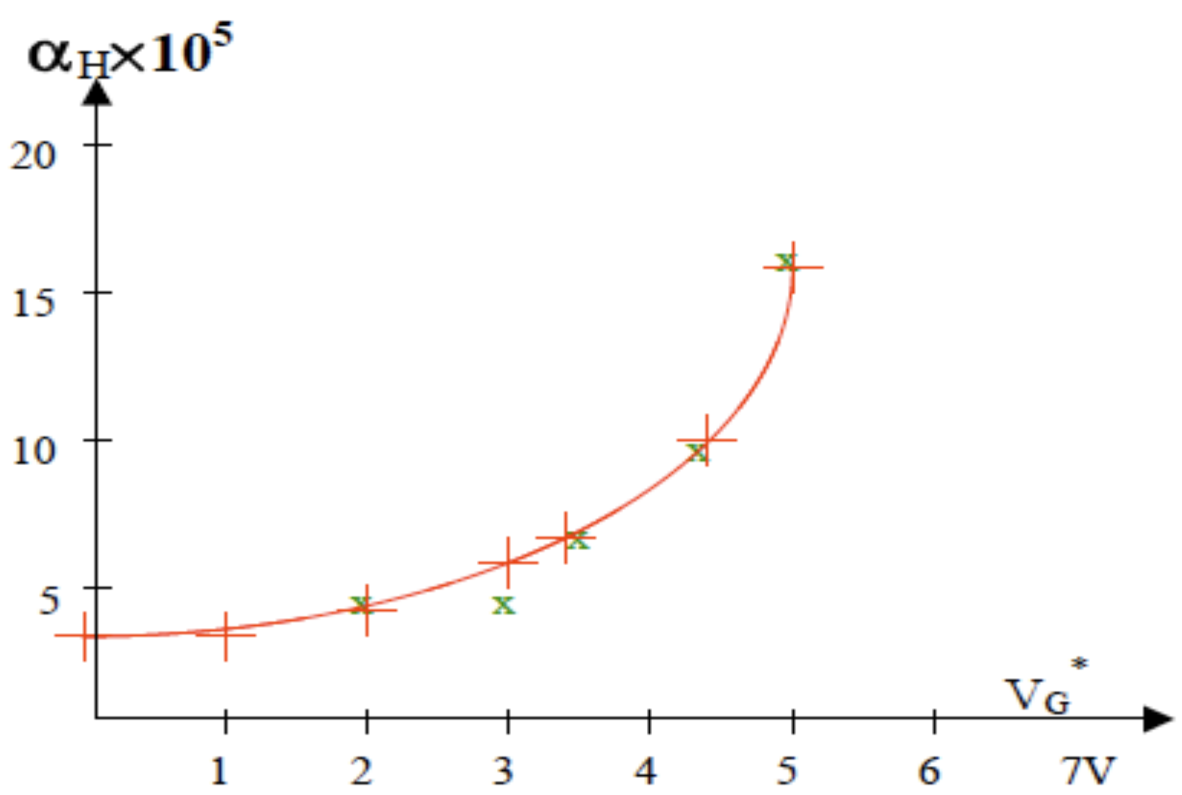

Fig. 2: Comparison of the quantum 1/f first-principles results with the experimental results **x** of Balandin et al., on "Flicker Noise in GaN/AlGaN Doped Channel Heterostructure FETs" IEEE-Electron Device Letters 19, 475-477 (1998) [31].

$$\alpha_{theor} = \frac{I_{ch}L^2}{qV_{DS}^2\mu^2}\left\{\frac{\alpha_H^{coh}I_{ch}}{qZ^2D_{eff}^2V_{th}V_{DS}} + \frac{\alpha_H^{conv}qv_s^2}{I_{ch}}\left(1-\frac{V_G^*}{V_{DS}}+\frac{V_{th}}{V_{DS}}\right)\Theta()\right\}$$

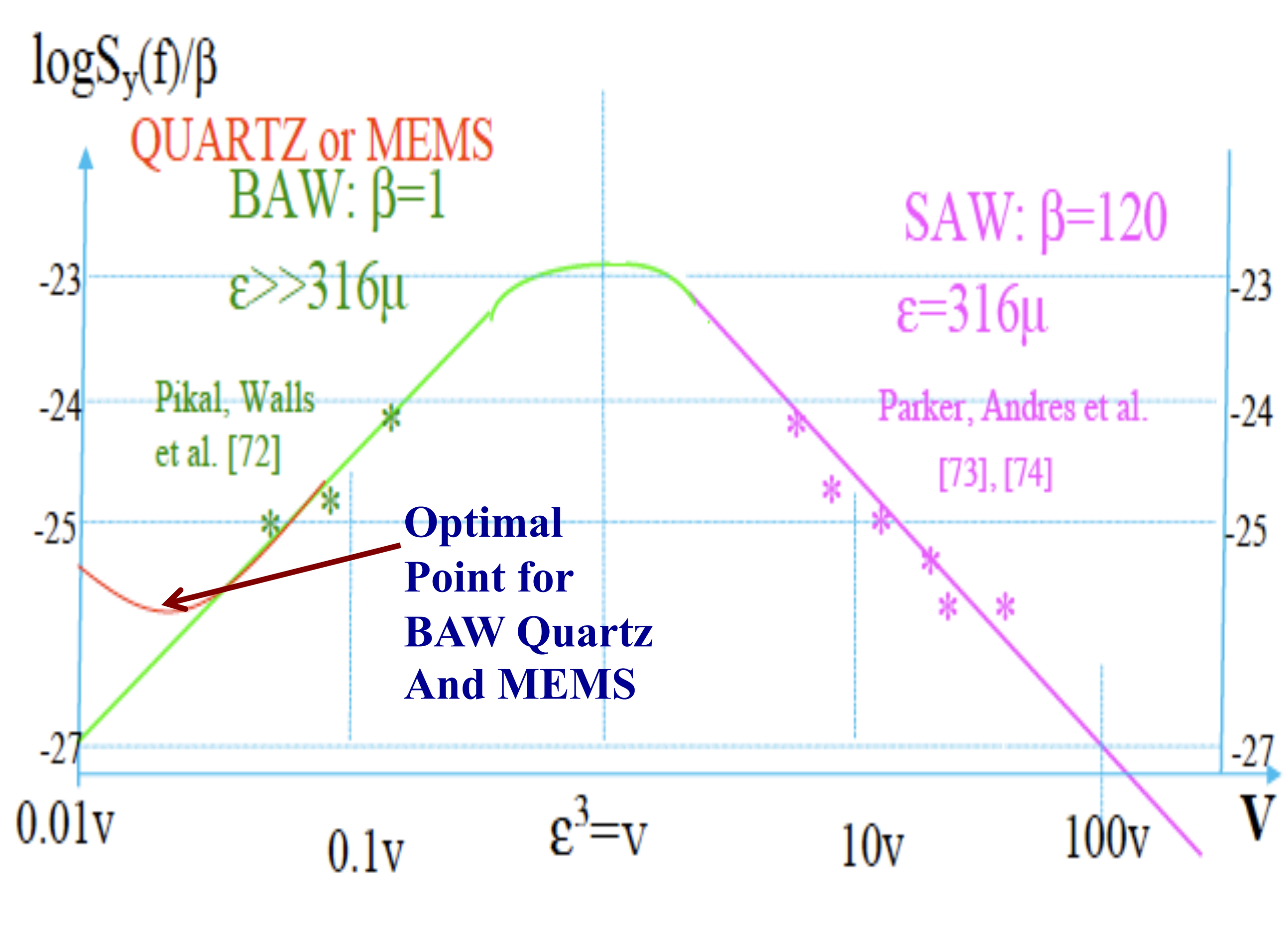

Fig. 3 Total FM noise in Si MEMS and quartz BAW & SAW resonators: the red line adds the classical white noise at small volumes.

Quantum 1/f Phase Noise in Resonators, Resonant Sensors and Oscillators

$$S_{\delta\sigma/\sigma}(f) = S_{\delta\Gamma/\Gamma}(f) = (4\alpha/3\pi f)(\Delta\mathbf{v}/c)^2 = 4\alpha(\Delta\dot{\mathbf{P}})^2/3\pi fe^2c^2$$

$S_{\delta\Gamma/\Gamma}(f) = 4\alpha(\Delta\dot{\mathbf{P}})^2/3\pi fe^2c^2$.

$W=n\hbar<\omega>=2(Nm/2)(dx/dt)^2$

$=(Nm/e^2)(edx/dt)^2=(m/Ne^2)g^2(\dot{\mathbf{P}}_i)^2$;

Applying a variation $\Delta n=1$ we get

$\Delta n/n=2|\Delta\dot{\mathbf{P}}|/|\dot{\mathbf{P}}|$, or $\Delta\dot{\mathbf{P}}=\dot{\mathbf{P}}/2n$.

$|\Delta\dot{\mathbf{P}}|=(N\hbar<\omega>/n)^{1/2}(e/2g)$.

Substituting this $\Delta\dot{\mathbf{P}}$ into the first Eq. above,

$\Gamma^{-2}S_\Gamma(f)=N\alpha\hbar<\omega>/3n\pi mc^2fg^2\equiv\Lambda/f$.

Using the harmonic oscillator relation

$\omega^2=\omega_o^2-2\Gamma^2$, $\omega\delta\omega=-2\Gamma\delta\Gamma$;

between resonance frequency ω and dissipation Γ, we obtain

$S_{\delta\omega/\omega}(f)=(1/4Q^4)S_{\delta\gamma/\gamma}(f)=(1/4Q^4)(\Lambda/f)$
$=N\alpha\hbar<\omega>/12n\pi mc^2fg^2Q^4$;

$S(f)=\beta'V/fQ^4$, for $V\le\varepsilon^3$,

and

$S(f)=\beta'\varepsilon^6/fVQ^4$, for $V\ge\varepsilon^3$,

Here, with an intermediary value $<\omega>=5\bullet10^9$/s, with $n=kT/\hbar<\omega>$, T=300K, $m=10^{-21}$gr. and $kT=4\bullet10^{-14}$ erg, we get
$\beta'=(N/V)\alpha\hbar<\omega>/12n\pi g^2mc^2=10^{22}(1/137)(10^{-27}5\bullet10^9)^2$
$/12kT\pi10^{-24}\ 9\ 10^{20}\cong1$.

Silicon MEMS Resonator Sensors

Magnetical excitation of a MEMS

Replacing the current change $e\Delta\mathbf{v}$ with $\Delta\dot{\mathbf{m}}$,

$S_{\delta\Gamma/\Gamma}(f)=(4\alpha/3\pi f)(\Delta\dot{\mathbf{m}}/ec)^2$.

The total mechanical energy $W=(n+1/2)\hbar\omega$ in the main oscillation mode of the resonator,

$(n+1/2)\hbar\omega=\kappa(\mathbf{m})^2/2$; $\hbar\omega=\kappa\dot{\mathbf{m}}\Delta\dot{\mathbf{m}}$.

$(\Delta\dot{\mathbf{m}})^2=(\hbar\omega/\kappa\dot{\mathbf{m}})^2=\hbar\omega/(2n+1)\kappa$.

$S_{\delta\Gamma/\Gamma}(f)=(4\alpha/3\pi f)\hbar\omega/(2n+1)\kappa(ec)^2$

$=4\omega/3\pi f(2n+1)\kappa c^3=2\hbar\omega^2/3\pi fW\kappa c^3$.

Electrical excitation of a silicon bar

$\hbar\omega=\kappa'\dot{\mathbf{p}}\Delta\dot{\mathbf{p}}$.

$(\Delta\dot{\mathbf{p}})^2=(\hbar\omega/\kappa'\dot{\mathbf{p}})^2=\hbar\omega/(2n+1)\kappa'$.

$S_{\delta\Gamma/\Gamma}(f)=(4\alpha/3\pi f)\hbar\omega/(2n+1)\kappa'(ec)^2$

$=4\omega/3\pi f(2n+1)\kappa'c^3=2\omega^2\hbar/3\pi fW\kappa'c^3$.

$W=M_{eff}v^2/2$. $\dot{\mathbf{p}}=qLv$,

With $M_{eff}=10^{-11}$g, $L=10^{-3}$cm and q=0.6 esu/cm, we obtain $\kappa'=2.7\cdot10^{-5}s^2cm^{-3}$. Substituting,

$S_{\delta\Gamma/\Gamma}(f)=(4\alpha/3\pi f)q^2L^2\hbar\omega/(2n+1)M_{eff}(ec)^2=$
$4q^2L^2\omega/3\pi f(2n+1)M_{eff}c^3=2q^2L^2\hbar\omega^2/3\pi fWM_{eff}c^3$.

$\omega=10^{12}s^{-1}$, and with $W=10^{-13}$ erg; we get $3\cdot10^{-18}/f$.

$S_{\delta\omega/\omega}=(1/4Q^4)S_{\delta\Gamma/\Gamma}$

$S_{\delta\omega/\omega}=(1/2Q^4)q^2L^2\hbar\omega^2/3\pi fWM_{eff}c^3$. (ES)

$S_{\delta\omega/\omega}=(1/2Q^4)(IL)^2\hbar\omega^2/3\pi fWM_{eff}c^5$ (Magn. Ex)

Electrostatically driven resonator is much 1/f -noisier than the magnetically driven.