Quantum 1/f Noise Theory and Experiment in QWIPs

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1/f Noise in the Dark Current of GaN QWIPs

- ✓ Conventional and Coherent Quantum 1/f effects
- ✓ Elementary application to homogeneous semiconductor samples
- ✓ Relation between Conventional and Coherent Quantum 1/f Effects
- ✓ Application to QWIPs
- ✓ Comparison with the experiment
- ✓ Conclusions

Quantum 1/f Formulas

Sources of Fundamental 1/f Noise in Devices & Sensors

Basic Quantum 1/f Formulas

1. The Quantum 1/f Effect (Q1/fE) is a fundamental fluctuation of all currents of charged particles j, of all scattering cross sections σ_s, recombination cross sections σ_r, tunneling rates or transition rates of any kind Γ, in short of all physical cross sections σ and process rates Γ, given by the universal formula

$$S_{\delta\sigma/\sigma}(f) = 4\alpha(\Delta v/c)^2/3\pi fN \qquad (1)$$

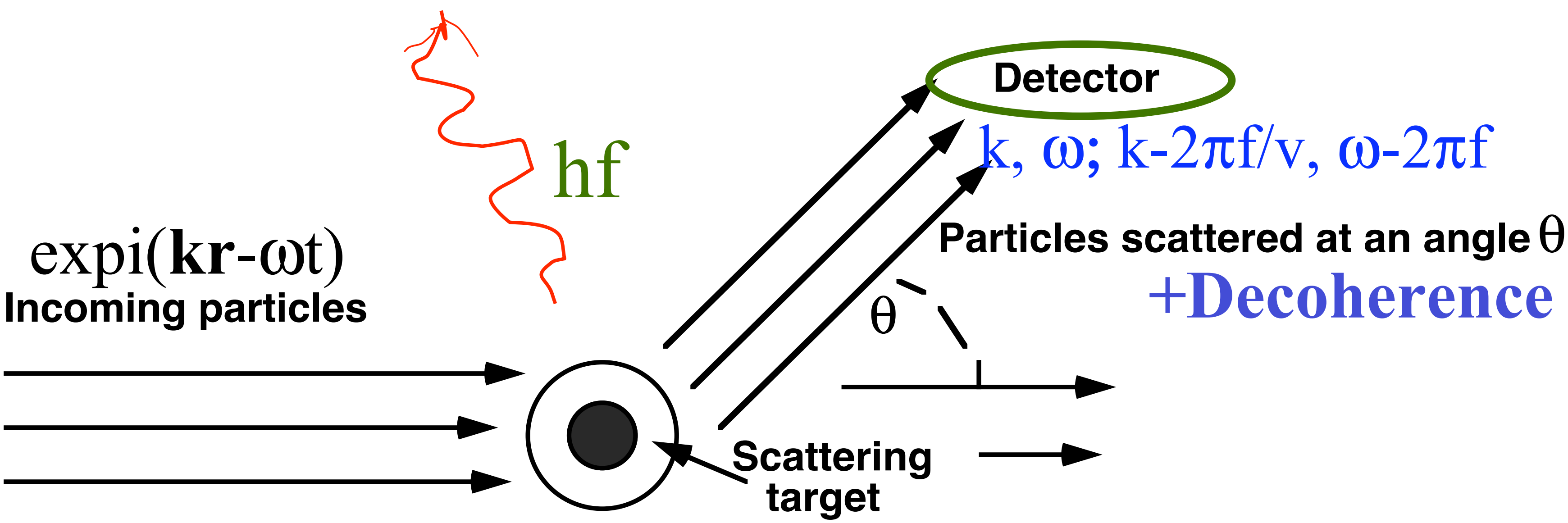
(**conventional quantum 1/f equation**) for small devices, and

$$S_{\delta j/j}(f) = 2\alpha/\pi fN = 4.6\ 10^{-3}/fN \qquad (2)$$

(coherent quantum 1/f equation) for large devices. These two Q1/fE forms can be combined into a single general formula, always valid. Here S(f) is the spectral density of fractional fluctuations in current, $\delta j/j$, in scattering or recombination cross section $\delta\sigma/\sigma$, or in any other process rate $\delta\Gamma/\Gamma$. $\alpha=2\pi e^2/hc =1/137$ is Sommerfeld's fine structure constant, a magic number of our world depending only on Planck's constant , the charge of the electron e and the speed of light in vacuum c. Finally, N is the number of particles defining j or σ.

Definition of the Quantum 1/f Effect
(conventional quantum 1/f noise)

2. The fundamental quantum 1/f fluctuation of physical cross sections and process rates caused by the emission of Bremsstrahlung (Handel, PRL 1975) is known as **Conventional Quantum 1/f Effect.** It is a fundamental infrared divergence phenomenon of QED, <u>a new aspect of quantum mechanics</u>. **Conv. Q1/fE + Coherent Q1/fE = Q1/f Noise**



Electromagnetic Conventional Quantum 1/f Noise

Elementary Derivation of Quantum 1/f Noise

3. Derivation of Conventional Quantum 1/f Noise

The scattered waves contain a part, of amplitude $b(f) = |b(f)|\exp(i\gamma)$, which lost energy due to emission of bremsstrahlung of any frequency f:

$\psi = (C/r)\exp(ikr - i\omega t)\,[\,1 + \int b(f)\exp(2\pi i f t)\,df\,]$; $\quad |b(f)| << 1$.

The scattered particle current density is

$j = (h/2mi)\,[\psi^*\nabla\psi - \psi\nabla\psi^*] = J + j'$, where $J \approx |C/r|^2 hk/m$ is the constant part;

$j' = 2|C/r|^2 \int (hk/m)\,|b(f)|\cos(2\pi f t + \gamma)\,df$ is the quantum fluctuation part of the current.

The power spectral density of the current fluctuations is the squared coefficient of the cosinus, divided by two:

$t \rightarrow (t - r/v)$

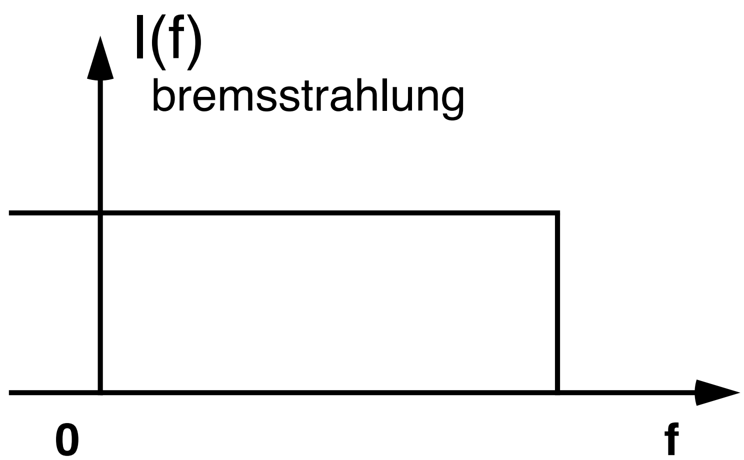
$$S_j(f) = 2\{|C/r|^2 (hk/m)\,|b(f)|\}^2$$

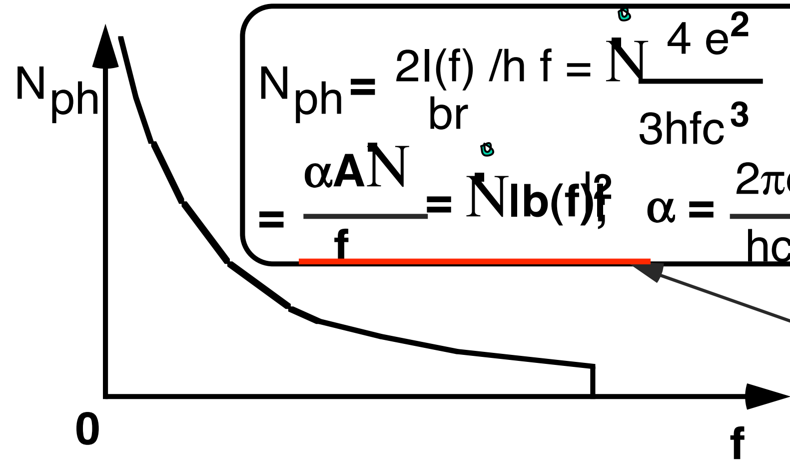
The spectral density of fractional fluctuations of the scattered current is obtained by dividing through J^2:

$$S_{j'/J}(f) = 2|b(f)|^2$$

The next viewgraph calculates b(f).

4. Elementary Derivation of Quantum 1/f Noise





$$N_{ph} = 2I(f)_{br}/hf = \dot{N}\frac{4e^2}{3hfc^3}\overline{(\Delta v)^2} = \frac{\alpha A \dot{N}}{f} = \dot{N}|b(f)|^2; \quad \alpha = \frac{2\pi e^2}{hc}; \quad A = \frac{2\overline{(\Delta v)^2}}{3\pi c^2}$$

Power radiated by an accelerated charge e

$$P = \frac{2e^2}{3c^3}\left[\frac{d\bar{v}}{dt}\right]^2; \quad \text{acceleration} = d\bar{v}/dt \approx \Delta\bar{v}\,\delta(t) = \Delta\bar{v}\int_{-\infty}^{\infty} e^{2\pi i f t}\,df$$

Total energy radiated by an accelerated charge in the scattering process:

$$W = \int P\,dt = \frac{2e^2}{3c^3}(\Delta\bar{v})^2 \int_{-\infty}^{\infty}dt\int_{-\infty}^{\infty}df\int_{-\infty}^{\infty}df' \exp[2\pi i(f-f')t] = \int_{-\infty}^{\infty}\frac{2e^2}{3c^3}(\Delta\bar{v})^2\,df$$

If $\dot{N}$ charges are scattered per time unit, the spectral density of emitted radiation power is

$$I(f)_{br} = \dot{N}\frac{2e^2}{3c^3}(\Delta\bar{v})^2;$$ this yields the framed result:

Elementary Derivation of Quantum 1/f Noise (Continued)

Substituting the expression of b(f) derived in the previous viewgraph, we obtain

$$S(f) = 2\,|b(f)|^2 = 2\alpha A/f.$$

If N particles or current carriers are simultaneously observed, they will contribute yield N independent noise contributions, while the current J also inceases N times; therefore we add a factor N in the numerator and two factors N in the denominator:

$$\boxed{S_{j'/J}(f) = \frac{2\alpha A}{fN}.}$$

The scattered current per unit incoming flux, however, is just what we call scattering cross section σ. Introducing the new notion of physical cross section σ whose expectation value is the conventional cross section, we get from $\delta j/j = j'/J = \delta\sigma/\sigma$:

$$\alpha A = \frac{2\alpha}{3\pi}\,\frac{\overline{(\Delta v)^2}}{c^2} << 1. \qquad S_{\delta\sigma/\sigma}(f) = \frac{2\alpha A}{fN}$$

This is the elementary, or simplified, derivation of quantum 1/f noise. Important aspects, such as the role of exchange between identical particles in the derivation, the role of infrared radiative corrections which change the exponent of f in the denominator from 1 to 1-αA if all orders are summed up, are ignored here.

Conventional Quantum 1/f Effect

- present in any cross section or process rate involving charged particles or current carriers
- Starting with the classical Lamor formula $P_e = 2q^2\mathbf{a}^2/3c^3$
- acceleration approximated by the delta function $\mathbf{a}(t) = \Delta v \delta(t)$
- one sided spectral density of emitted bremsstrahlung power $4q^2(\mathbf{\Delta v})^2/3c^3$
- the probability amplitude of photon emission is therefore $[4q(\Delta v)^2/3hfc^3]^{1/2}e^{i\gamma}$

- Let ψ be a representative Schrodinger wave function of scattered outgoing particles; **Decoherence randomizes all phases γ**
- Spectral density given by the product of the squared probability amplitude of photon emission (proportional to 1/f) and the squared non-bremsstrahlung amplitude (independent of f)

- The resulting spectral density of fractional probability density fluctuations is therefore

$$|\psi|^{-4}S_{|\psi|2}(f) = 8q^2(\Delta \mathbf{v})^2/3hfNc^3 = 2\alpha A/fN = j^{-2}S_j(f) \qquad (2.1)$$

- The spectral density can also be derived in second quantization, using the commutation rules for boson field operators.

- For fermions one repeats the calculation replacing in the derivation the commutators of field operators by anticommutators, which yields

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/f(N-1) \qquad (2.2)$$

- Nonlinearity is the source of the 1/f spectrum in both the classical and quantum form of the theory; the quantum 1/f effect is an infrared divergence phenomenon, this divergence being the result of the same nonlinearity.

- The quantum 1/f effect is, in fact, the first time-dependent infrared radiative correction; it is also deterministic in the sense of a well determined wave function, once the initial phases γ of all field oscillators are given

Elementary applications to homogeneous semiconductor samples

- Consider a homogeneous sample of length L, cross section A, volume V=AL, carrier mobility μ, carrier concentration n and total number of carriers N=nAL,

- The conductance C=nμeA/L and the resistance R=1/C will exhibit quantum 1/f fluctuations with a spectral density $S_{\delta C/C}$ of fractional fluctuations δC/C = -δR/R given by

$$S_{\delta C/C}(f) = S_{\delta R/R}(f) = S_{\delta\mu/\mu}(f) = \alpha_o/fN \qquad (2.3)$$

- To calculate α_o, first evaluate the parameter

  $s = nA.5.5.10^{-13}$ cm

- If $s \geq 1$, coherent quantum 1/f noise is observed with

  $\alpha_o = 2\alpha/\pi$.

- If $s \ll 1$, $(\alpha_o)_{conv}$ is calculated from the equations (2.1) or (2.2) above for each type of scattering which limits the mobility $\mu = e\tau/m^*$ of the carriers, where τ is the mean collision time or scattering time of the carriers and m^* is their effective mass.

- In general, we have for any s value : (**Any size device**)

- $\boxed{(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN]}$

Conventional Q 1/f effect in the mobility

- Matthiessen' rule allows us to write, in terms of mobility,

$$1/\mu = \sum_j 1/\mu_j \tag{2.4}$$

- Applying a quantum 1/f fluctuation, squaring and averaging quantum mechanically and statistically, we obtain as a reasonable first approximation

$$\delta\mu/\mu^2 = \sum_j \delta\mu_j / \mu_j^2 \;\; ; \mathbf{S_{\delta\mu/\mu}(f) =} \sum_j (\mu/\mu_j)^2 \, \mathbf{S_{\delta\mu/\mu j}(f)} \tag{2.5}$$

- The factors $(\mu/\mu_j)^2$ gauge the importance of each of the scattering processes, such as umklapp, inter-valley, phonon or impurity scattering, in their ability to limit the resultant mobility.

Impurity scattering

- In the case of impurity scattering, n_i ,one obtains

$$S_{\delta\mu/\mu}(f) = S_{\delta\sigma/\sigma}(f)$$

- The physical scattering cross section **σ**, in turn, exhibits the Q1/fE with the spectral density given by Eqs. (2.1) and (2.2)

$$S_{\delta\sigma/\sigma}(f)_i = (4\alpha/3\pi fN)<[\mathbf{\Delta v}/c]^2>\approx(3.10^{-3}/fN)(\hbar\Delta\mathbf{k}/m^*c)^2 \qquad (2.6)$$

- The average quadratic velocity change of the electrons in a scattering process is smaller in impurity scattering than in lattice scattering which includes normal phonon scattering, inter-valley scattering in indirect band gap semiconductors with several "valleys", and umklapp scattering, as well as optical phonon scattering.

- The Coulomb/Rutherford or Conwell/Weiskopf scattering cross section is proportional to $1/|\Delta \mathbf{k}|^4$, which favors small angle scattering.

- There are a few larger angle scattering events which are most effective in limiting the mobility and which therefore are decisive in the exact evaluation of the Q1/fE coefficient as a slow function of n_i, the concentration of impurities

- This corresponds approximately to assuming randomizing collisions,

$$<(\mathbf{\Delta v})^2>_i = 2<v^2> = 6k_B T/m^* \qquad (2.7)$$

although impurity scattering is not randomizing

- With m_o representing the free electron mass, we obtain

$$S_{\delta\mu/\mu}(f)_i = S_{\delta\sigma/\sigma}(f)_i = (4\alpha/3\pi f N)<[6k_B T/m^*c^2]> \approx (10^{-9}/fN)(Tm_o/4m^*100K) \quad (2.8)$$

The quantum 1/f noise power present in impurity scattering is therefore proportional to T.

Normal acoustic phonon scattering

- For normal phonon scattering, the product $\sigma v n_i$ is replaced by the lattice scattering rate Γ, given by an effective number of phonons times the squared phonon scattering matrix element, $|M|^2$.

- The latter exhibits quantum 1/f fluctuations, because the carriers emit bremsstrahlung photons in the scattering process

- If the mobility μ of electrons (of random velocity $v=k/m^*$) is limited by phonon scattering, then

$$S_{\delta\mu/\mu}(f)_{ap}=S_{\delta\Gamma/\Gamma}(f)_{ap}=(4\alpha/3\pi fN)<(\hbar\Delta\mathbf{k}/m^*c)^2>=(3.10^{-3}/fN)<(\hbar\Delta\mathbf{q}/m^*c)^2> \quad (2.9)$$

- Using the linear approximation of the acoustic phonon dispersion relation $E_q=v_s q\hbar$, with v_s denoting the speed of sound, we obtain for a thermal phonon with $E_q=k_BT/2$

$$<(\hbar\Delta\mathbf{q}/m^*c)^2> = (k_BT/2v_sm^*c)^2 = 1.25.10^{-5}\ (m_o/m^*)^2 \qquad (2.10)$$

- We finally obtain

$$\mathbf{S_{\delta\mu/\mu}(f)_{ap} = (3.75.10^{-8}/fN)(m_o/m^*)^2} \qquad (2.11)$$

- A more rigorous treatment for the many types of scattering present in semiconductors, taking into account the corrections introduced by the momentum distribution of the carriers and the phonon distribution function at the temperature T is discussed later for the case of silicon.

Umklapp scattering

- In this case the momentum change is close to the smallest reciprocal lattice vector approximated by $\hbar G=2\pi\hbar/a$, where a is the lattice constant

- Therefore, the first form of Eq. (2.10) yields

$(\alpha_{oU})_{conv} = (4\alpha/3\pi)[2\pi\hbar/am^*c]^2$;

$$\mathbf{S_{\delta\mu/\mu}(f)_U = (6.10^{-8}/fN)(m_o/m^*)^2} \qquad (2.12)$$

Inter-valley scattering

- In indirect band gap semiconductors the location of the energy minima of the conduction band in k-space is different from the location of the valence band energy maxima

- In thermal equilibrium electrons and holes are present close to the minima of the conduction and valence bands.

- Scattering processes carrying electrons from one minimum (or valley) to the other are known as inter-valley scattering. This is large-angle scattering, compared with normal (intra-valley) scattering, and it is therefore affected by larger conventional quantum 1/f noise, almost as large as Umklapp scattering.

- For g-processes which scatter an electron to the valley symmetrically located on the other side of the origin, Eq. (2.12) remains valid with a correction factor of (0.85)

- On the other hand for f-processes in Si, scattering electrons between to neighboring valleys, the factor is 2 times smaller

- There is also the possibility of inter-valley scattering with umklapp which requires a correction factor of $(1\text{-}0.85)^2$.

  Eq. (2.12) thus yields for inter-valley scattering with Umklapp

$$(\alpha_{oIU})_{conv}=0.0225(4\alpha/3\pi)[2\pi\hbar/am^*c]^2$$

$$\mathbf{S_{\delta\mu/\mu}(f)_{IU} =(1.35.10^{-9}/fN)(m_o/m^*)^2} \qquad (2.13)$$

Physical derivation of the coherent quantum 1/f effect

- This effect arises in a beam of electrons (or other charged particles propagating freely in vacuum) from the definition of the physical electron as a bare particle plus a coherent state of the electromagnetic field

- It is caused by the energy spread characterizing any coherent state of the electromagnetic field oscillators, an energy spread which spells non-stationarity, i.e., fluctuations

- To find the spectral density of these inescapable fluctuations which are known to characterize any quantum state which is not an energy eigenstate, we use an elementary physical derivation based on Schrödinger's definition of coherent states

- The coherent quantum 1/f effect will be derived in three steps:
  - first we consider a hypothetical world with just one single mode of the electromagnetic field coupled to a beam of charged particles; considering the mode to be in a coherent state, we calculate the autocorrelation function of the quantum fluctuations in the particle-density (or concentration) which arise from the non-stationarity of the coherent state.
  - Then we calculate the amplitude with which this one mode is represented in the field of an electron, according to electrodynamics.
  - Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step

- Let a mode of the electromagnetic field be characterized by the wave vector **q**, the angular frequency $\omega = cq$ and the polarization λ. We write the coherent state of amplitude $|z_q|$ and phase arg z_q in the form

$$|z_q\rangle = \exp[-(1/2)|z_q|^2]\exp[z_q a_q^+]\,|0\rangle$$

$$= \exp[-(1/2)|z_q|^2][\ \sum_{n=0}^{\infty} z_q^n/\sqrt{n!}\ ]\ |n\rangle \qquad (2.14)$$

- We use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n\rangle = (2^n n!\sqrt{\pi})^{-1/2} \exp[-x^2/2]\, H_n(x)\, e^{in\omega t} \qquad (2.15)$$

- This yields for the coherent state $|z_q>$ the representation

$$\psi_q(x) = \exp[-(1/2)|z_q|^2]\exp[-x^2/2] \sum_0^\infty \frac{\left[Z_q e^{i\omega t}\right]^n H_n(x)}{\left[n!\left(2^n\sqrt{\pi}\right)\right]^{1/2}}$$

$$=\exp[-|z_q|^2/2]\exp[-x^2/2]\exp[-z_q{}^2 e^{-2i\omega t} + 2xz_q e^{i\omega t}] \qquad (2.16)$$

- The corresponding autocorrelation function of the probability density function, obtained by averaging over the time t or the phase of z_q, is, for $|z_q|<<1$,

$$P_q(\tau,x)=<|\psi_q|_t{}^2|\psi_q|_{t+\tau}{}^2>=\{1+8x^2|z_q|^2[1+\cos\omega\tau] - 2|z_q|^2\}\exp[-x^2/2] \qquad (2.17)$$

- Integrating over x from $-\infty$ to ∞, we find the autocorrelation function

$$A^1(\tau) = (2)^{-1/2}\{1 + 2|z_q|^2\cos\omega\tau\} \qquad (2.18)$$

- We now determine the amplitude z_q with which the field mode q is represented in the physical electron. Letting the bare particle dress itself through its interaction with the electromagnetic field, i.e. by performing first order perturbation theory with the interaction Hamiltonian

$$H' = A_\mu j^\mu = -(e/c)\mathbf{v}\bullet\mathbf{A} + e\phi \qquad (2.19)$$

- Another way is to Fourier expand the electric potential e/r of a charged particle in a box of volume V. In both ways we obtain the well-known result

$$|z_q|^2 = \pi(e/q)^2(\hbar cqV)^{-1} \qquad (2.20)$$

- Considering now all modes of the electromagnetic field, we obtain from the single - mode result of Eq. (2.18)

$$A(\tau)= C\ \Pi_q\{1 + 2|z_q|^2\cos \omega q\tau\} = C\{1 + \Sigma_q\ 2|z_q|^2\cos \omega q\tau\}$$
$$= C\{1 + 4(V/2^3\pi^3) \int d^3q\ |z_q|^2\cos \omega q\tau\} \qquad (2.21)$$

- Using Eq. (2.23) we obtain

$$A(\tau) = C\{1 + 4\pi(V/2^3\pi^3)(4\pi/V)(e^2/\hbar c) \int(dq/q)\cos \omega_q\tau\}$$
$$= C\{1 + 2(\alpha/\pi) \int\cos(\omega\tau)d\omega/\omega\} \qquad (2.22)$$

- The first term in curly brackets is unity and represents the constant background, or the d.c. part of the current carried by the beam of particles through vacuum.

- The autocorrelation function for the relative (fractional) density fluctuations, or for the current density fluctuations in the beam of charged particles is obtained therefore by dividing the second term in curly brackets by the first term

- According to the Wiener-Khintchine theorem, the coefficient of $\cos\omega\tau$ is the spectral density of the fluctuations, $S_{|\psi|2}$ for the particle concentration, or S_j for the current density

$$\mathbf{j = e(k/m)|\psi|^2}$$

$$S_{|\psi|2}<|\psi|^{-2}> = S_j<j>^{-2} = 2(\alpha/\pi f N) = 4.6 \bullet 10^{-3}\, f^{-1} N^{-1} \qquad (2.23)$$

- This result is in general larger than the conventional Quantum 1/f Effect introduced above and can also be derived directly from the well-known physical QED propagator of Kulish, Faddeev and Zwanziger.

Relation between coherent and conventional quantum 1/f effect

- The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers.

- In very small samples or electronic devices, the magnetic energy E_m, per unit length, of the current carried by the sample is written

$$E_m = \int(B^2/8\pi)d^3x = [nevS/c]^2[1/4 + \ln(R/r)] \qquad (2.24)$$

which is much smaller than the total kinetic energy E_k of the drift motion of the individual carriers

$$E_k = \Sigma mv^2/2 = nSmv^2/2 = E_m/s \qquad (2.25)$$

- The "coherence ratio" is defined as

$$s = E_m/E_k = 2ne^2S/mc^2[1/4 + \ln(R/r)]$$
$$\approx 2e^2N'/mc^2 = 2N'r_o = 5.68\cdot 10^{-13}\mathrm{cm}\cdot N' \qquad (2.26)$$

where $N' = nS$ is the number of carriers per unit length of the sample, $r_o = e^2/mc^2 = 2.84\cdot 10^{-13}$ cm is the classical radius of the electron, and the expression in rectangular brackets has been approximated by unity in the last form.

- We thus expect the total observed spectral density of the mobility fluctuations to be approximated by a relation of the form (as indicated before and framed in red)

$$\mathbf{(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN]} \qquad (2.27)$$

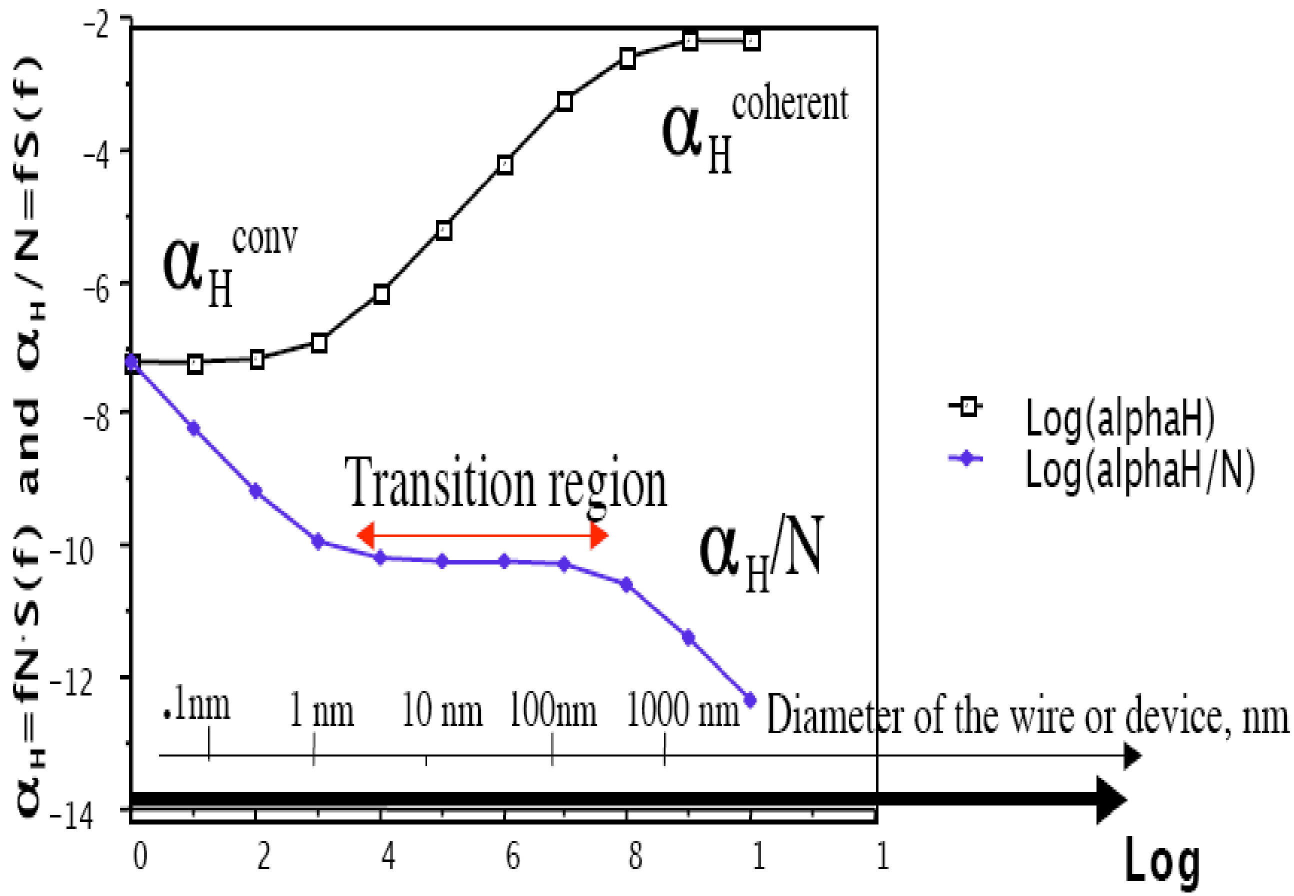


Fig. 1: The Q1/f parameter α_H and spectrum of fractional mobility fluctuations $fS_{\delta\mu/\mu} \equiv \alpha_H/N$ shown as a function of the number N of carriers in the sample.

The case of FETs/HFETS

- Modern FETs and HFETs try to increase the denominator N in Eq. (2.27) without markedly increasing the coherence parameter s, by choosing an extreme case of a rectangular cross section for the portion between source and drain, and even more extreme for the channel, which is the region under the gate
- The channel is reduced to a 2-dimensional quantum sheet, which is a quantum well of size (thickness) t in the 3rd dimension, towards the gate
- Most important, the "length" L_{DS} of the source-to-drain distance is only of the order of 1micron, while the gate length is just about a quarter micron or less

- In this case the kinetic energy E_k per unit length in the direction of L_{DS} is still given by Eq. (2.25), but the magnetic energy E_m per unit length in the direction of L_{DS} is roughly proportional with the first power of w only, instead of w^2, and can be approximated by

$$E_m = \pi[\ln(w/2L_{DS})]L_{DS}[nevS/c]^2/w \qquad (2.28)$$

- This indicates decoherence along the device width w. For $w \gg L_{DS} > t$, this yields a coherence ratio

$$s \approx \pi n r_o t L_{DS} \ln(w/2L_{DS}) \qquad \text{for } w \gg L_{DS} > t, \qquad (2.29)$$

which shows that only an effective width $w=w_{eff}$ about equal to L_{DS} should be used in the calculation of the coherence parameter s in this special case; larger widths are subject to de-coherence, i.e., their magnetic energy no longer increases proportional to w^2, or N'^2

DERIVATION OF QUANTUM 1/f NOISE

1.1.3 Derivation of the Conventional Quantum 1/f Effect

We start with the expression of the Heisenberg representation state |f> of N identical bosons of mass M emerging at an angle θ from some scattering process with undetermined bremsstrahlung energy losses reflected in their one-particle waves $\varphi_i(\xi_i)$

$$|\mathbf{f}\rangle = (N!)^{-1/2}\, \Pi_i \int d^3\xi_i \varphi_i(\xi_i)\psi^+(\xi_i)\, |\mathbf{0}\rangle = \Pi_i \int d^3\xi_i \varphi_i(\xi_i)\, |\mathbf{f^o}\rangle, \qquad (1)$$

where $\psi^+(\xi_i)$ is the field operator creating a boson with position vector ξ_i and |0> is the vacuum state, while $|f^o\rangle$ is the vacuum field state with N bosons of position vectors ξ_i with i = 1...N. All products and sums in this paper run from 1 to N, unless otherwise stated.

DERIVATION OF QUANTUM 1/f NOISE

To calculate the particle density autocorrelation function in the outgoing scattered wave, we need the expectation value of the operator

$$O(\mathbf{x}_1,\mathbf{x}_2) = \psi^+(\mathbf{x}_1)\psi^+(\mathbf{x}_2)\psi(\mathbf{x}_2)\psi(\mathbf{x}_1), \qquad (2)$$

known as the operator of the pair correlation. Using the commutation properties of the boson field operators, we first calculate the matrix element

$$N!\langle S^o|O|f^o\rangle=$$

$$\Sigma'_{\mu\nu}\Sigma'_{mn}\delta(\boldsymbol{\eta}_\nu-\mathbf{x}_1)\delta(\boldsymbol{\eta}_\mu-\mathbf{x}_2)\delta(\boldsymbol{\xi}_n-\mathbf{x}_1)\delta(\boldsymbol{\xi}_m-\mathbf{x}_2)\Sigma_{(i,j)}\Pi'_{ij}\delta(\boldsymbol{\eta}_j-\boldsymbol{\xi}_i). \qquad (3)$$

Here the prime excludes $\mu=\nu$ and $m=n$ in the summations and excludes $i=m$, $i=n$, $j=\mu$ and $j=\nu$ in the product. The summation $\Sigma_{(i,j)}$ runs over all permutations of the remaining N-2 values of i and j.

DERIVATION OF QUANTUM 1/f NOISE

On this basis we now calculate the complete matrix element

$$<f|O|f> = [1/N(N-1)]\ \Sigma'_{\mu\nu}\Sigma'_{mn} \int d^3\eta_\mu \int d^3\eta_\nu \int d^3\xi_m \int d^3\xi_n$$

$$\varphi_\mu{}^*(\boldsymbol{\eta}_\mu)\varphi_\nu{}^*(\boldsymbol{\eta}_\nu)\varphi_m(\boldsymbol{\xi}_m)\varphi_n(\boldsymbol{\xi}_n)\delta(\boldsymbol{\eta}_\nu-\mathbf{x}_1)\delta(\boldsymbol{\eta}_\mu-\mathbf{x}_2)\delta(\boldsymbol{\xi}_n-\mathbf{x}_1)\delta(\boldsymbol{\xi}_m-\mathbf{x}_2)$$

$$= [1/N(N-1)]\ \Sigma'_{\mu\nu}\Sigma'_{mn}\ \varphi_\mu{}^*(\mathbf{x}_2)\varphi_\nu{}^*(\mathbf{x}_1)\varphi_m(\mathbf{x}_1)\varphi_n(\mathbf{x}_2) \qquad (4)$$

The one-particle states are spherical waves emerging from the scattering center located at x=0:

$$\varphi(\mathbf{x}) = (C/x)e^{iKx}\ [1 + \Sigma_{\mathbf{k}l} b(\mathbf{k},l)e^{-iqx}\ a^+_{\mathbf{k},l}]. \qquad (5)$$

Here C is an amplitude factor, K the boson wave vector magnitude, b(**k**,l) the bremsstrahlung amplitude for photons of wave vector **k** and polarization l, while $a^+_{\mathbf{k},l}$ is the corresponding photon creation operator, allowing the photon state to be created from the vacuum.

DERIVATION OF QUANTUM 1/f NOISE

The momentum magnitude loss $q=Mck/\hbar K \equiv M2\pi f/\hbar K$ is necessary for energy conservation in the Bremsstrahlung process. Substituting Eq. (6) into Eq. (4), we obtain

$$\langle f|O|f\rangle = |C/x|^4\{N(N-1) + 2(N-1)\Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2[1+\cos q(x_1-x_2)]\}, \quad (6)$$

where we neglected a small term of higher order in $b(\mathbf{k},l)$. To perform the angular part of the summation in Eq. (6), we calculate the current expectation value of the state in Eq. (5), and compare it to the well known cross section without and with bremsstrahlung

$$\mathbf{j} = (h\mathbf{K}/Mx^2)[1 + \Sigma_{kl}|b(k,l)|^2] = \mathbf{j}_o[1 + \int \alpha A df/f], \quad (7)$$

where the quantum fluctuations have disappeared, where $\alpha=e^2/\hbar c$ is the fine structure constant, $\alpha A=(2\alpha/3\pi)(\Delta\mathbf{v}/c)^2$ is the fractional bremsstrahlung rate coefficient, also known in QED as the infrared exponent.

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DERIVATION OF QUANTUM 1/f NOISE

Here $\Delta\mathbf{v}$ is the velocity change $\hbar(\mathbf{K}-\mathbf{K}_o)/M$ of the scattered boson, and $f=ck/2\pi$ the photon frequency. Eq. (14) gives

$$\langle f|O|f\rangle = |C/x|^4\{N(N-1) + 2(N-1)(\int \alpha A df/f)[1+\cos q(x_1-x_2)]\}, \quad (8)$$

which is the pair correlation function, or density autocorrelation function along the scattered beam with $df/f=dq/q$. The spatial distribution fluctuations along the scattered beam will also be observed as fluctuations in time at the detector, at any frequency f. According to the Wiener-Khintchine theorem, we obtain the spectral density of fractional scattered particle density ρ, (or current j, or cross section σ) fluctuations in frequency f or wave number q by dividing the coefficient of the cosine by the constant term N(N-1):

$$\boldsymbol{\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/fN} \quad (9)$$

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1.2 Derivation of Coherent Quantum 1/f Noise

$$-i\langle\Phi_o|T\psi_{s'}(x')\psi_s|\Phi_o\rangle \equiv \delta_{ss'}\,G_s(x'-x)$$
$$= (i/V)\Sigma_{\mathbf{p},s}\mathrm{expi}[\mathbf{p}(\mathbf{r}-\mathbf{r}')-\mathbf{p}^2(t-t')/2m]/\hbar\}\,n_{\mathbf{p},s}\ \{-i\mathbf{p}(\mathbf{r}-\mathbf{r}')/\hbar$$
$$+i(m^2c^2+\mathbf{p}^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}. \qquad (1)$$

$$\langle\Phi_o|T\psi_{s'}\psi_s(x)\psi_s\psi_{s'}(x')|\Phi_o\rangle$$
$$= \langle\Phi_o|\psi_s\psi_s(x)|\Phi_o\rangle\langle\Phi_o|\psi_{s'}\psi_{s'}(x')|\Phi_o\rangle$$
$$- \langle\Phi_o|T\psi_{s'}(x')\psi_s|\Phi_o\rangle\langle\Phi_o|T\psi_s(x)\psi_{s'}|\Phi_o\rangle. \qquad (2)$$

$$n/2 = N/2V = \langle\Phi_o|\psi_s\psi_s(x)|\Phi_o\rangle$$

$$A_{ss'}(x-x') \equiv \langle\Phi_o|T\psi_s\psi_{s'}\psi_{s'}(x')\psi_s(x)|\Phi_o\rangle$$
$$=(n/2)^2 + \delta_{ss'}\,G_s(x'-x)G_s(x-x'). \qquad (3)$$

Derivation of Coherent Quantum 1/f Noise

$$A(x-x') = 1 - (1/n^2)G_s(x-x')G_s(x'-x)$$

$$= 1 - (1/N^2)\Sigma_{\mathbf{p},s}\Sigma_{\mathbf{p}',s}\exp i[(\mathbf{p}-\mathbf{p}')(\mathbf{r}-\mathbf{r}')-(\mathbf{p}^2-\mathbf{p}'^2)(t-t')/2m]/\hbar\}n_{\mathbf{p},s}n_{\mathbf{p}',s}$$

$$\times\{\mathbf{p}(\mathbf{r}-\mathbf{r}')/\hbar -(m^2c^2+\mathbf{p}^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}$$

$$\times\{\mathbf{p}'(\mathbf{r}-\mathbf{r}')/\hbar -(m^2c^2+\mathbf{p}'^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}. \qquad (4)$$

The energy and momentum differences between terms of different **p** are large, leading to rapid oscillations in space and time which contain only high-frequency quantum fluctuations. The low-frequency and low-wavenumber part A_l of this relative density autocorrelation function is given by the terms with $\mathbf{p}=\mathbf{p}'$

$$A_l(x-x') = 1 - (1/N^2)n^2_{\mathbf{p},s}\{\mathbf{p}(\mathbf{r}-\mathbf{r}')/\hbar -(m^2c^2+\mathbf{p}^2)^{1/2}(t-t')(c/\hbar)\}^{2\alpha/\pi} \qquad (5)$$

Derivation of Coherent Quantum 1/f Noise

$$\approx 1 - (1/N)|\mathbf{p}_o(\mathbf{r}-\mathbf{r}')/\hbar - mc^2\tau/\hbar|^{2\alpha/\pi} \quad \text{for } p_F \ll |p_{o3} - mc^2\tau/z|. \quad (6)$$

$\tau \equiv t-t'$, with $z \equiv |\mathbf{r}-\mathbf{r}'|$. Using a well known identity, with arbitrarily small cutoff ω_o, we obtain from Eq. (23) with $\theta \equiv |\mathbf{p}_o(\mathbf{r}-\mathbf{r}')/\hbar - mc^2\tau/\hbar|$

$$A_l(x-x') = 1 + [(2\alpha/\pi N)^{2\alpha/\pi} \cos(\theta\omega) d\omega/\omega] \times \{\cos\alpha + (2\alpha/\pi)^{2n-2\alpha/\pi}[(2n)!(2n-2\alpha/\pi)]^{-1}\}^{-1}. \quad (7)$$

$$S_{\delta j/j}(k) \approx [2\alpha/\pi\omega N][2\pi mc^2/\hbar\omega]^{2\alpha/\pi} \approx 2\alpha/\pi\omega N = 0.00465/\omega N. \quad (8)$$

1.3 Basic Quantum 1/f Formulas, Connection Conv/Coherent

We derived Quantum 1/f Noise as a fundamental fluctuation of all currents of charged particles j, of all scattering cross sections σ_s, recombination cross sections σ_r, tunneling rates or transition rates of any kind Γ, in short of all physical cross sections σ and process rates Γ, given by the universal formula

$$S_{\delta\sigma/\sigma}(f) = 4\alpha(\Delta \mathbf{v}/c)^2/3\pi f N \qquad (1)$$

(conventional quantum 1/f equation) for small devices, and

$$S_{\delta j/j}(f) = 2\alpha/\pi f N = 4.6\ 10^{-3}/fN \qquad (2)$$

(coherent quantum 1/f equation) for large devices. These two Q1/f Effect forms can be combined into a single general Q1/f Noise formula, always valid. Here S(f) is the spectral density of fractional fluctuations in current, $\delta j/j$, in scattering or recombination cross section $\delta\sigma/\sigma$, or in any other process rate $\delta\Gamma/\Gamma$. $\alpha = e^2/\hbar c = 1/137$ is Sommerfeld's fine structure constant, a magic number of our world depending only on Planck's constant , the charge of the electron e and the speed of light in vacuum c. Finally, N is the number of particles defining j or σ.

Basic Quantum 1/f Formulas, Conv/Coherent

The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy E_m, per unit length of the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int (B^2/8\pi) d^3x = [nevS/c]^2 \ln(R/r) \qquad (3)$$

is much smaller than the total kinetic energy E_k of the drift motion of the individual carriers

$$E_k = \Sigma mv^2/2 = nSmv^2/2 = E_m/s. \qquad (4)$$

Here we have introduced the magnetic field B, the carrier concentration n, the cross sectional area S and radius r of the cylindrical sample (e.g., a current carrying wire), the radius R of the electric circuit, and the "coherence ratio"

$$s = E_m/E_k = 2ne^2S/mc^2 \ln(R/r) \approx 2e^2N'/mc^2 = 2N'r_o, \qquad (5)$$

Basic Quantum 1/f Formulas, Conv/Coherent

where $N' = nS$ is the number of carriers per unit length of the sample and the natural logarithm $\ln(R/r)$ has been approximated by one in the last forms. Here $r_o = 2.84\ 10^{-13}$ cm is the classical radius of the electron. In general, thus, $s = 5.69 \cdot 10^{-13}$ cm N', i.e., the number of carriers in a very thin sample slice of about 5.7 fermi thickness. We expect the observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN] \qquad (6)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side

$$\alpha_H = [1/(1+s)]\alpha_{Hconv} + [s/(1+s)]\alpha_{Hcoh}. \qquad (7)$$

In these equations $\alpha A = 2\alpha(\Delta\mathbf{v}/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum 1/f effect.

Basic Quantum 1/f Formulas, Conv/Coherent

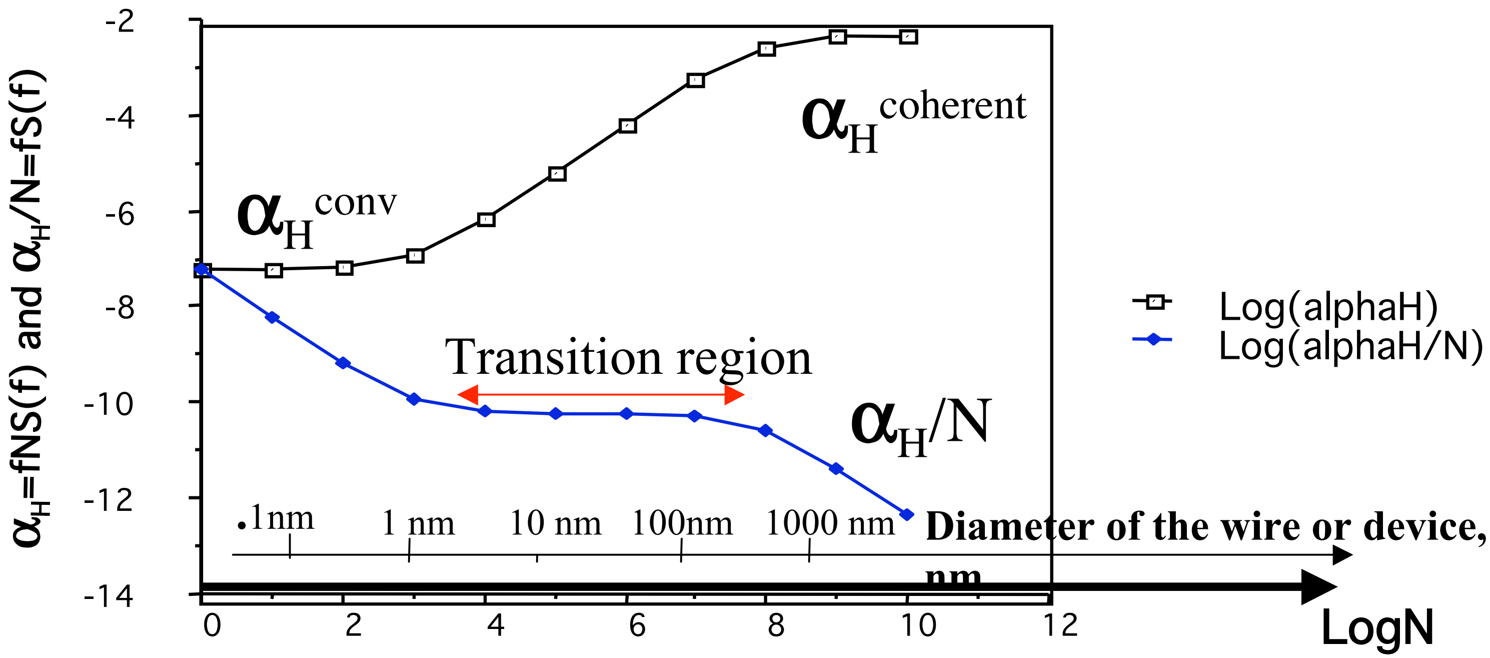


Fig. 1: The Q1/f parameter α_H and **spectrum of fractional mobility fluctuations $fS_{\delta\mu/\mu} \equiv \alpha_H/N$** we predict the proximity effect, is marked by red arrow. The wire diameter scale shown as a function of the number N of carriers in the sample. The transition region, in which gets shifted to the right (or left) proportionally when the concentration of carriers increases (or decreases) from the assumed value of 10^{17} cm^{-3}. We took the cross section to be ($140nm^2$), the length 5 mm, and the concentration n = $10^{17}cm^{-3}$ as an example.

1.6 Basic Quantum 1/f Formulas and Numerical α^{conv} Calculation

The *coherent Q1/f coefficient* is applicable only for large samples or devices:

$$\alpha_H^{coh} = 2\alpha/\pi = 4.6\ 10^{-3} \qquad (1)$$

The *conventional quantum 1/f coefficient*

$$\alpha_{Hi}^{conv} = (4\alpha/3\pi)\langle(\Delta\mathbf{v}/c)^2\rangle_i \qquad (2)$$

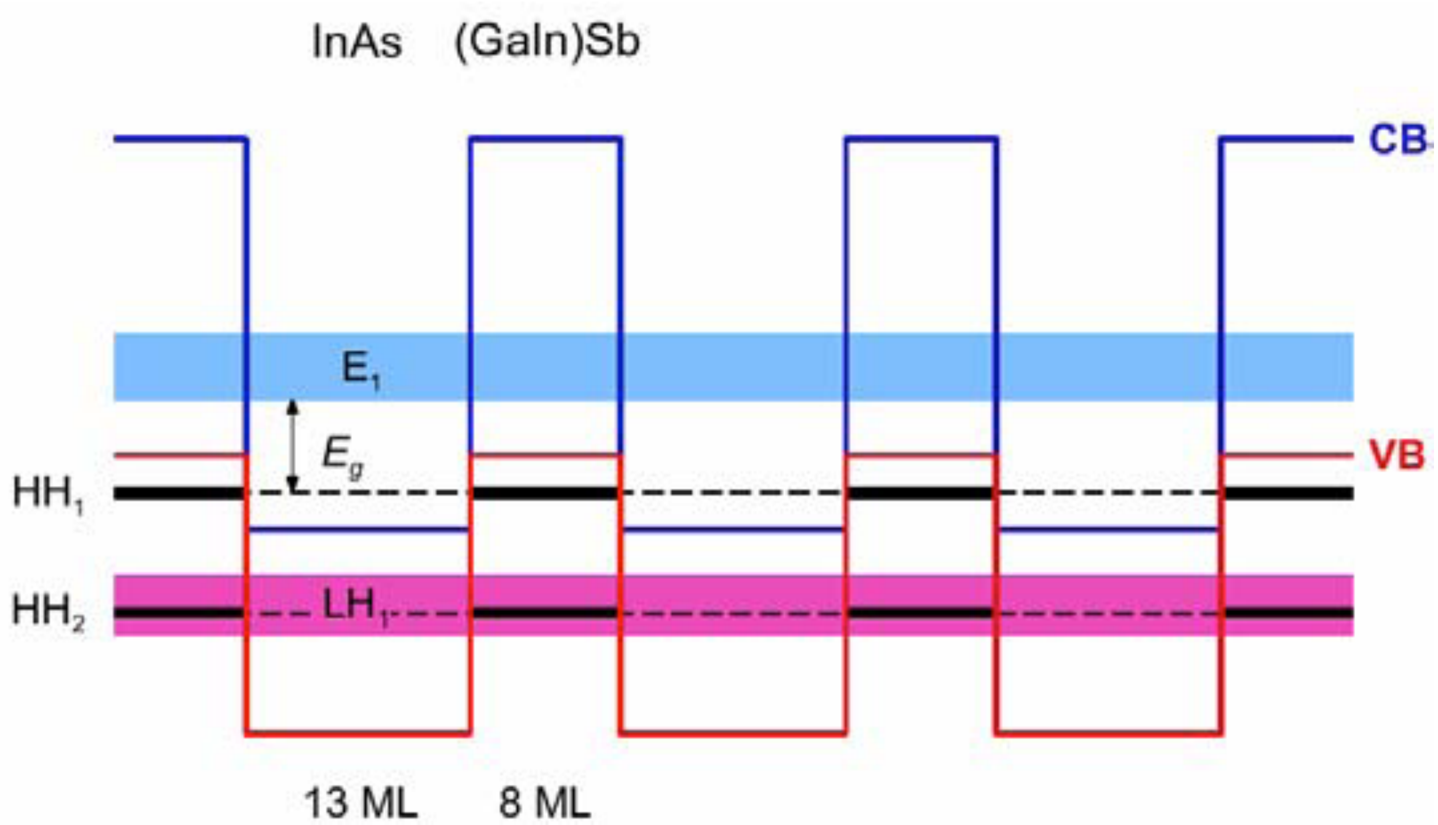
is calculated as a function of temperature for each type "i" of scattering resulting in a limiting mobility μ_i if no other scattering process was present. It is finally calculated also for the resulting, or total, mobility μ from the formulas:

2. QUANTUM WELL INTERSUBBAND PHOTODETECTORS

Quantum well intersubband photodetectors (QWIPs) yield a dark current that is always present, even without illumination and background radiation. Fluctuations of this dark current will compete with the signal to be detected and limit the detectivity. A first calculation of fundamental 1/f noise in the QWIPS dark current, based on the universal quantum 1/f formulas, is presented and compared with experimental data on GaInAs/InP QWIPs.

We will investigate the noise associated with this current. The noise is described by the conventional quantum 1/f effect [4], [6], [12] which describes the fundamental fluctuations in the physical cross sections and process rates that generate the dark current. The calculation of the 1/f noise is then compared to experimental data from Jutao Jiang (and Manijeh Razeghi) [13]. A similar calculation was performed earlier by Handel for n+p junction infrared detectors and MIS infrared detectors.

QUANTUM WELL INTERSUBBAND PHOTODETECTORS



Band structure of a InAs/GaInSb superlatice

Application to QWIPs

- The Quantum Well Intersubband Photodetector is a semiconduc-
- tor photon detector that responds to absorption within its bandgaps.

- The structure is designed to allow an excited carrier within the well to escape by thermal activation or tunneling, joining others to form what is called the dark current.

- The noise is described by the conventional quantum 1/f effect which describes the fundamental fluctuations in the physical cross sections and process rates that generate the dark current. The calculation of the 1/f noise is then compared to experimental data.

1.Theory

- Applying an electric field to the quantum well accelerates the electrons, and the resulting drift velocity gives rise to the dark current. Conservation of energy gives an expression for this velocity

$$\frac{m}{2}(\Delta v)^2 = (L_b + L_w)eE + |W_0| \qquad (3.1)$$

- According to Eq. (2.1), the spectral density of these current fluctuations is given by

$$S_{\frac{\delta\rho}{\rho}} = S_{\frac{\delta j_{dark}}{j_{dark}}} = \frac{4\alpha}{3\pi fN}\left(\frac{\Delta v}{c}\right)^2 = j^{-2} S_{\delta j_{dark}}(f) \qquad (3.2)$$

- Substituting the velocity change from Eq. (3.1), we obtain

$$S_{\delta j_{dark}} = \frac{4\alpha}{3\pi fN}\left(\frac{2}{mc^2}\right)\left[\left(L_b + L_w\right)eE + W_0\right] j_{dark}^2 \tag{3.3}$$

- Next, recalling the relation between the dark particle-current j_{dark} and the number of carriers

$$j_{dark} = \left\langle \frac{dN}{dx} \right\rangle \left\langle v_{carriers} \right\rangle \tag{3.4}$$

- Classically, an electron accelerated between two points has a final velocity of

$$v_{end} = \sqrt{2a\left(\Delta l\right)} \qquad (3.5)$$

- The average velocity of the carriers is

$$\left\langle v_{carriers} \right\rangle = \tfrac{1}{2}\sqrt{2a\left(\Delta l\right)} = \tfrac{1}{2}\sqrt{2a\left(L_b + L_w\right)} \quad (3.6)$$

- To simplify, we let $L = L_b + L_w$ represent the lattice constant of the quantum well and use $a = eE/m$ for the acceleration

- The dark current is then

$$j_{dark} = \frac{N}{L}\frac{1}{2}\sqrt{\frac{2eEL}{m}} \qquad (3.7)$$

- Finally, with this relation between the number of carriers and the dark current, the spectral density can be simplified

$$S_{\delta j_{dark}} = \frac{8\alpha}{3\pi fmc^2}\left(\frac{\sqrt{eE}}{\sqrt{2Lm}}\right)\left[LeE + W_0\right]j_{dark} \qquad (3.8)$$

- To quantify the dark current fluctuations, we now apply the data and parameters of GaInAs/InP QWIPs

Table 1. Values of the GaInAs/InP QWIP Parameters

PARAMETER:	Value used	
Dark Current j_{dark} at 77K	2	μA
MQW lattice period L	570	Å
Total voltage applied 20EL	2	V
Voltage applied on 1 well = EL	0.1	V
Binding energy in the well W_o	0.1	eV

- Next, to apply the specific parameters of the QWIP device to our derived spectral density equation (3.8), using a binding energy, W_0, for the well is 0.251eV. At low temperatures (around 70K), the dark current is around 2μA.

- Dividing Eq. (3.8) by the square of the current j_{dark}, we obtain the spectral density of its fractional fluctuations in the form, with the effective mass of the electrons in the InP barrier $m=0.077m_o$

$$S_{\delta j/j}(f)=(j_{dark})^{-2}S_{\delta j}=(2.25\cdot 10^{-9}/f)(1.66\cdot 10^{12})(8\cdot 10^{-14})48=1.44\cdot 10^{-8}/f \qquad (3.10)$$

- Amplifying this with the square of $j_{dark}=1.25\cdot 10^{13}/s$ we obtain the spectral density of dark current fluctuations

$$S_{\delta j}(f)=(2.25\cdot 10^{-9}/f)(1.66\cdot 10^{12})(1.25\cdot 10^{13}/s^2)=(2.26\cdot 10^{18}/s^2)/f \qquad (3.11)$$

- The corresponding power spectral density $S_{\delta I}(f)$ of electrical current fluctuations δI, rms noise current I_n and detectivity D* are

$$S_{\delta I}(f)=(1.6\cdot 10^{-19}C)^2(4.7\cdot 10^{16}/s^2)/f=5.73\cdot 10^{-20}A^2/f$$

and

$$I_n=2.4\cdot 10^{-10}\ A/f^{1/2}; \qquad D^*=\frac{R}{I_{noise}}=\frac{R\sqrt{f}}{2.4\cdot 10^{-10}A} \qquad (3.12)$$

2. Comparison with the experiment

- Experimental values for the current noise (I_{noise}) is obtained by means of the detectivity and responsivity data reported by Jutao Jiang. The NEP=P_{equiv} is the optical power producing a signal equal with the rms noise current. The detectivity (D^*) is proportional with the inverse of the noise equivalent optical power NEP=P_{equiv},

$$D^* = \frac{1}{P_{equiv}} \qquad (3.13)$$

- Responsivity (R) is defined as the current generated per optical power unit

$$R = \frac{I_{noise}}{P_{equiv}} \qquad (3.14)$$

- This implies that the current noise can be written

$$I_{noise} = \frac{R}{D^*} \qquad \text{or, equivalently,}$$

$$\log I_{noise} = \log R - \log D^* \tag{3.15}$$

- The data gives the following relations for detectivity and responsivity, both as functions of the bias voltage

$$\log D^* = 9 - \frac{V}{2volts} \tag{3.16}$$

$$\log R = -2 + \frac{3V}{4volts} \tag{3.17}$$

- These expressions can easily be compared with Eq. (3.12):

$$\log I_{noise} = \log R - \log D^{*}$$
$$= \left(-2 + \frac{3V}{4volts}\right) - \left(9 - \frac{V}{2volts}\right) = -11 + \frac{5V}{4volts} \qquad (3.18)$$

- For V=2 volts this experimental result is within one order of magnitude and just slightly larger than our theoretical result of $2.4 \cdot 10^{-10}$ A/$f^{1/2}$ obtained in Eq. (3.12) above from first principles, without any adjustable parameters

- In the Q1/f theory result (3.8) above, the last 3 factors in the spectral density of current fluctuations are proportional to $V^{1/2}$, V, and V respectively, resulting in a total proportionality to $V^{2.5}$ for V not too small, in nice agreement with the experiment.

- In general, the collisions will also be important. To investigate the collision-dominated case, we will consider the drift velocity of the charge carriers to be $\nu_d = \mu E$, where μ is the mobility of the charge carrier. The spectral density is given by

$$S_{\frac{\delta\rho}{\rho}} = S_{\frac{\delta J_{dark}}{J_{dark}}} = \frac{4\alpha}{3\pi fN}\left(\frac{eEL_p + W_0}{m^* c^2}\right) = j^{-2} S_{\delta J_{dark}}(f) \qquad (3.19)$$

- where total dark current is $J_{dark} = n_o A \nu_d = n_o A \mu E$. The number of charge carriers N can be written as $n_o AL$ where n_o is the doping level of the well. L is the total length of the device and L_p is the length of the periodicity, or the length of the well and barrier. The effective mass, m^*, is related to the mass of the free particle by $m^* = 0.077 m_0$.

- The spectral density is then

$$S_{\frac{\delta J_{dark}}{J_{dark}}} = \frac{4\alpha}{3\pi f n_0 A L}\left(\frac{eEL_p + W_0}{m^* c^2}\right) \qquad (3.20)$$

This yields results similar to Eq. (3.12)

Quantum 1/f Fluctuations in the Dark Current:

Excited carriers within the well escape with a rate Γ affected by quantum 1/f fluctuations, joining others to form what is called the dark current. The quantum fluctuations of this rate show a 1/f spectral density of fractional fluctuations given by

$$S_{\frac{\delta\rho}{\rho}} = S_{\frac{\delta j_{dark}}{j_{dark}}} = \frac{4\alpha}{3\pi fN}\left(\frac{\Delta v}{c}\right)^2 = j^{-2} S_{\delta j_{dark}}(f)$$

where $\mathbf{\Delta v}$ is the vector change in velocity of the charge carriers in the scattering process and N is the number of particles that defines the scattered current $\mathbf{j}$.

Here, the case of thermionic emission of carriers is the only process considered; the dark current is then composed of the number of carriers excited out of the quantum wells and the number of carriers in the continuum, or conduction band region of the device, which are swept along by the applied bias, and eventually reach the collector. Because of the independence, the squared average is also independent and contains no cross terms.

The number of carriers defining the dark current depends on several factors including well volume, doping concentration, applied bias, in addition to the ionization (W_i) and activation (W_0) energies of the donors. For each QWIP device, an expression for the spectral density of dark current fluctuations can be formulated based on the activation energy, the formulation of the number of carriers (N) in the dark current, and the applied bias (U)[1].

$$\left\langle S_{\delta I_d} \right\rangle_f = \left\langle \left(\delta I_d\right)^2 \right\rangle = \frac{4\alpha}{3\pi f N}\left(\frac{2W_0 + 2eU}{m^* c^2}\right) I_d^2$$

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Comparison with Experiment:

Theoretical calculations of dark current noise are then compared with several reported experimental findings. The first comparison is a $Ga_{0.47}\,In_{0.53}\,As$ /InP quantum well QWIP fabricated and tested by J.Jiang[2]. Using the parameters of Jiang's QWIP listed below in Table 1, a reasonable approximation of the activation and ionization energies was formulated, leading to an expression for the number of carriers in the dark current. The spectral density of dark current noise fluctuations can then be calculated for increasing values of dark current and applied bias. The results of the comparison are graphically presented in Figure 1 below.

PARAMETER	VALUE
MQW periods	20
Quantum well thickness ($Ga_{0.47}In_{0.53}As$)	57 Å
Barrier thickness (InP)	514 Å
Well doping level	5×10^{17} cm^{-3}
Effective electron mass (InP)	0.077 m_o
Area of QWIP mesa	400 x 400 μm
Conduction band offset	0.251 eV

Jiang's $Ga_{0.47}In_{0.53}As$ /InP quantum well structure parameters

Bias (Volts)	Dark Current (Amps)	Experimental Noise Current ($A / Hz^{1/2}$)	Theoretical Noise Current ($A / Hz^{1/2}$)
0.5	5.0×10^{-8}	1.8×10^{-12}	1.1×10^{-12}
1.0	2.5×10^{-7}	2.4×10^{-12}	5.5×10^{-12}
1.5	8.0×10^{-7}	3.2×10^{-12}	1.8×10^{-11}
2.0	2.0×10^{-6}	9.5×10^{-12}	4.4×10^{-11}
2.5	4.0×10^{-6}	1.8×10^{-10}	9.1×10^{-11}
3.0	1.0×10^{-5}	3.5×10^{-10}	2.2×10^{-10}
3.5	2.0×10^{-5}	4.9×10^{-10}	3.9×10^{-10}
4.0	3.5×10^{-5}	2.1×10^{-9}	7.8×10^{-10}

Experimental and Theoretical noise current as a function of bias voltage for Jiang's $Ga_{0.47}In_{0.53}As$ /InP QWIP.

Bias (Volts)	Dark Current (Amps)	Experimental Noise Current (A / Hz$^{1/2}$)	Theoretical Noise Current (A / Hz$^{1/2}$)
0.5	5.0×10^{-8}	1.8×10^{-12}	1.1×10^{-12}
1.0	2.5×10^{-7}	2.4×10^{-12}	5.5×10^{-12}
1.5	8.0×10^{-7}	3.2×10^{-12}	1.8×10^{-11}
2.0	2.0×10^{-6}	9.5×10^{-12}	4.4×10^{-11}
2.5	4.0×10^{-6}	1.8×10^{-10}	9.1×10^{-11}
3.0	1.0×10^{-5}	3.5×10^{-10}	2.2×10^{-10}
3.5	2.0×10^{-5}	4.9×10^{-10}	3.9×10^{-10}
4.0	3.5×10^{-5}	2.1×10^{-9}	7.8×10^{-10}

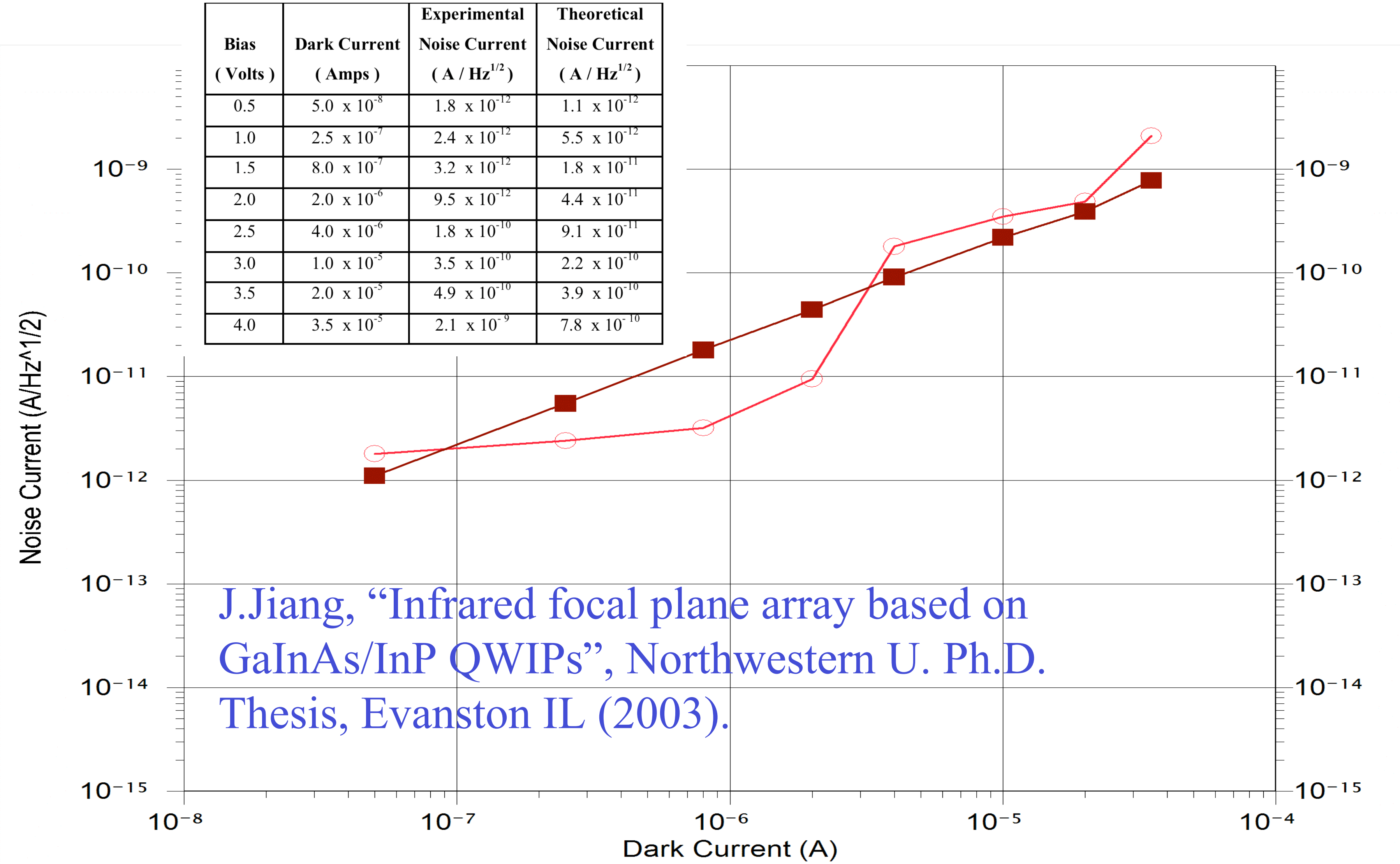

FIGURE 1: Experimental and theoretical noise current vs. dark current for the case of Jiang's $Ga_{0.47}In_{0.53}As$ /InP QWIP. The thin line represents experimental data values. Squares represent theoretical values.

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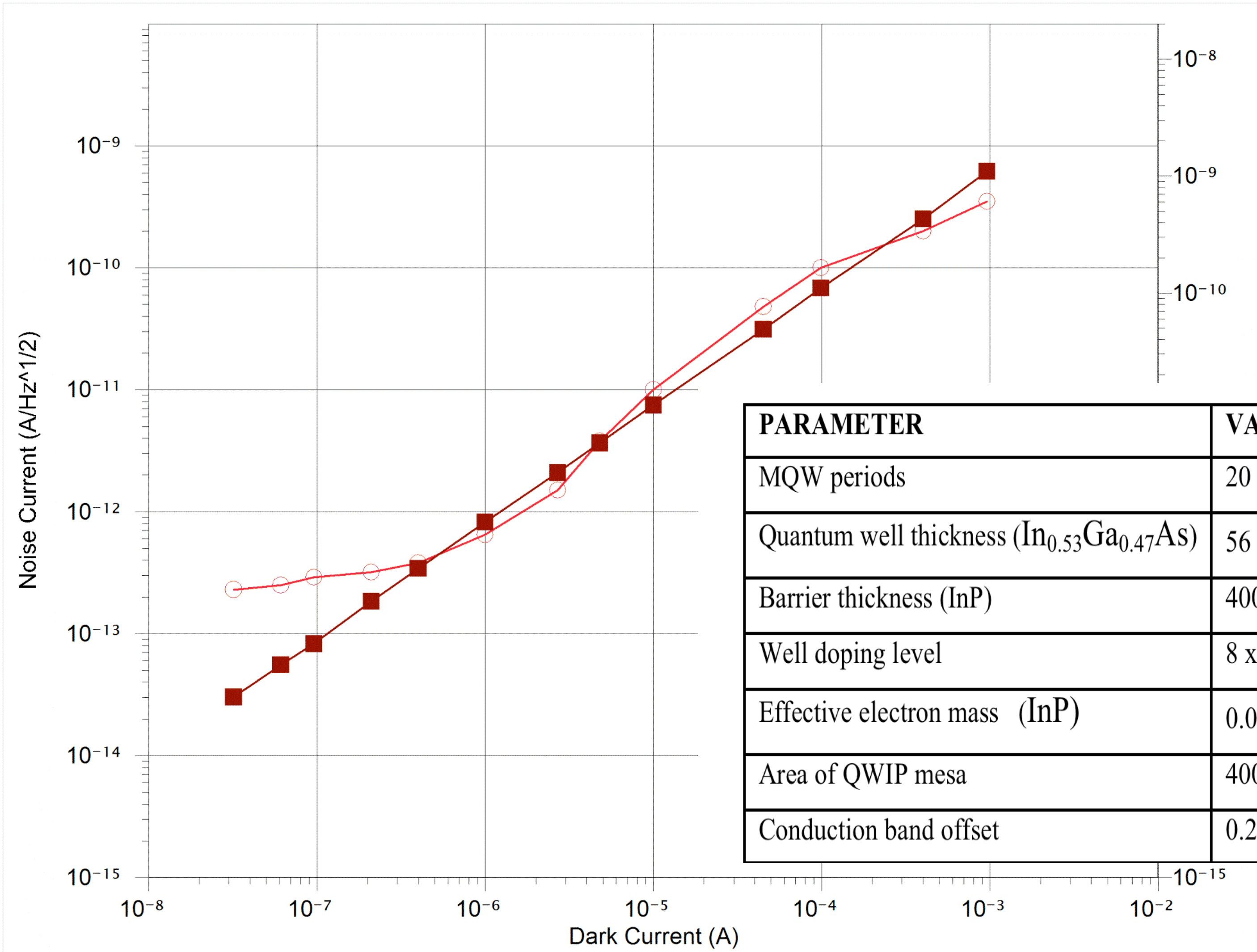
Samples of Jelen & Thibaudeau

The comparisons with two more experiments are displayed below. Two samples of Jelen[3] and two samples of Thibaudeau[4]. Sample 'A' of Jelen is a $In_{0.53}Ga_{0.47}As/InP$ structure and sample 'B" is a $In_{0.52}Ga_{0.38}Al_{0.1}As$ /InP QWIP structure. The sample of Thibaudeau is a $GaAs/Al_{0.263}Ga_{0.737}As$ QWIP.

In each case, the theoretically calculated noise spectrum are shown to agree with experimental findings, for various types of QWIP structures and molar concentrations.

BIAS (Volts)	DARK CURRENT (Amps)
0.01	3.2×10^{-8}
0.10	6.1×10^{-8}
0.20	9.6×10^{-8}
0.45	2.1×10^{-7}
0.80	4.0×10^{-7}
1.20	1.0×10^{-6}
1.50	2.7×10^{-6}
1.70	4.8×10^{-6}
2.00	1.0×10^{-5}
2.50	4.5×10^{-5}
2.90	9.9×10^{-5}
3.50	4.0×10^{-4}
4.00	9.6×10^{-4}

Dark current measured as a function of bias voltage for Jelen's $In_{0.53}Ga_{0.47}As/InP$ QWIP sample 'A'



PARAMETER	VALUE
MQW periods	20
Quantum well thickness ($In_{0.53}Ga_{0.47}As$)	56 Å
Barrier thickness (InP)	400 Å
Well doping level	8×10^{17} cm^{-3}
Effective electron mass (InP)	0.077 m_o
Area of QWIP mesa	400 x 400 μm
Conduction band offset	0.229 eV

Experimental and theoretical noise current vs. dark current for the case of Jelen's $In_{0.53}Ga_{0.47}As$/InP QWIP sample 'A'. The thin line represents experimental data values. Squares represent theoretical values.

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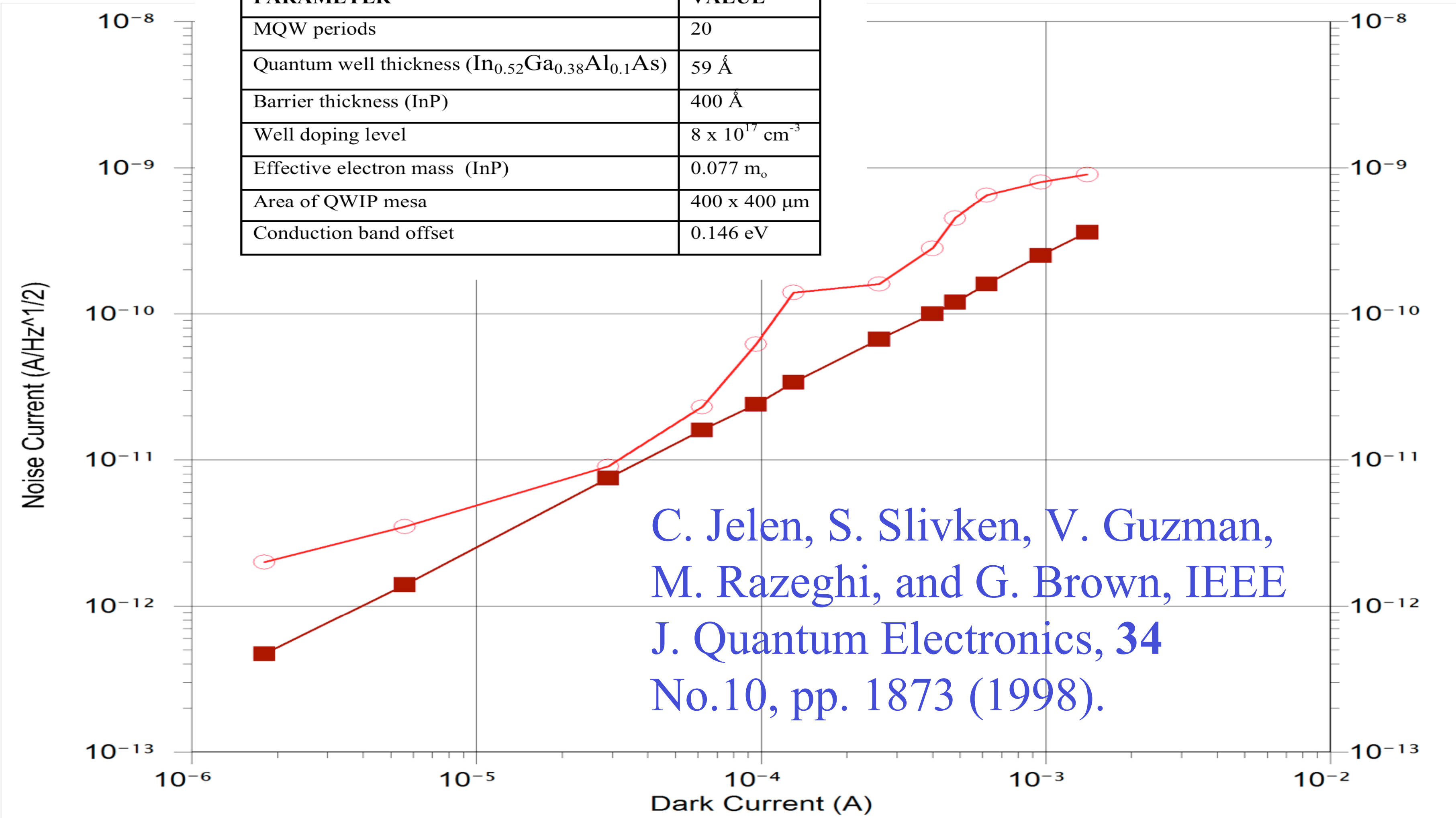
Bias (Volts)	Dark Current (Amps)	Experimental Noise Current ($A / Hz^{1/2}$)	Theoretical Noise Current ($A / Hz^{1/2}$)
0.01	1.8×10^{-6}	2.0×10^{-12}	4.7×10^{-13}
0.10	5.6×10^{-6}	3.5×10^{-12}	1.4×10^{-12}
0.26	2.9×10^{-5}	9.0×10^{-12}	7.5×10^{-12}
0.50	6.2×10^{-5}	2.3×10^{-11}	1.6×10^{-11}
0.72	9.6×10^{-5}	6.2×10^{-11}	2.4×10^{-11}
1.00	1.3×10^{-4}	1.4×10^{-10}	3.4×10^{-11}
1.40	2.6×10^{-4}	1.6×10^{-10}	6.7×10^{-11}
1.60	4.0×10^{-4}	2.8×10^{-10}	1.0×10^{-10}
1.80	4.8×10^{-4}	4.5×10^{-10}	1.2×10^{-10}
2.00	6.2×10^{-4}	6.5×10^{-10}	1.6×10^{-10}
2.20	9.6×10^{-4}	8.0×10^{-10}	2.5×10^{-10}
2.50	1.4×10^{-3}	9.0×10^{-10}	3.6×10^{-10}

Experimental and theoretical noise current as a function of bias voltage for Jelen's $In_{0.52}Ga_{0.38}Al_{0.1}As$ /InP QWIP sample 'B'.

PARAMETER	VALUE
MQW periods	20
Quantum well thickness ($In_{0.52}Ga_{0.38}Al_{0.1}As$)	59 Å
Barrier thickness (InP)	400 Å
Well doping level	8×10^{17} cm^{-3}
Effective electron mass (InP)	0.077 m_o
Area of QWIP mesa	400 x 400 μm
Conduction band offset	0.146 eV

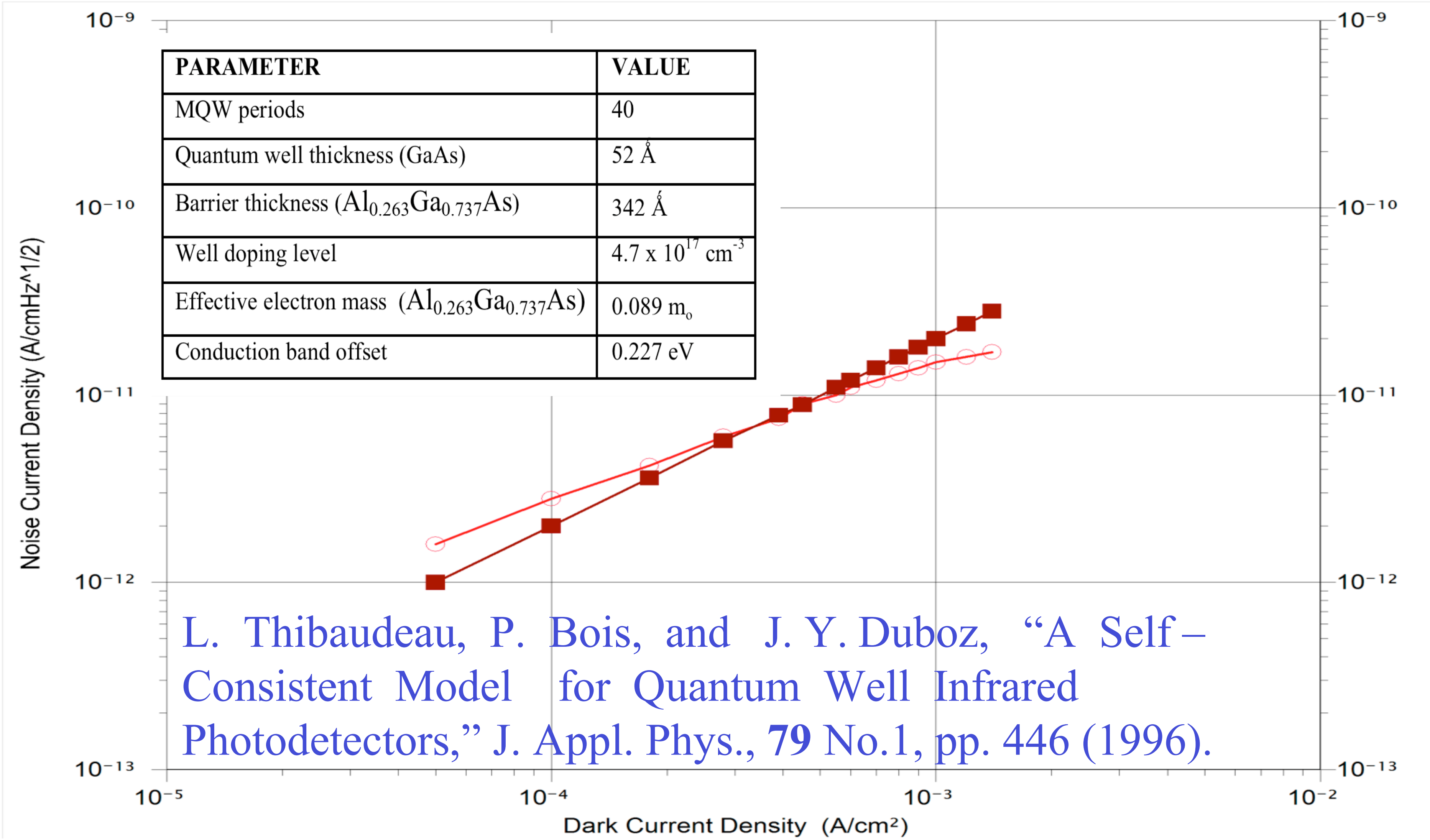


C. Jelen, S. Slivken, V. Guzman, M. Razeghi, and G. Brown, IEEE J. Quantum Electronics, **34** No.10, pp. 1873 (1998).

Experimental and theoretical <u>noise current vs. dark current</u> for the case of Jelen's $In_{0.52}Ga_{0.38}Al_{0.1}As$ /InP QWIP sample 'B'. The thin line represents experimental data values. Squares represent theoretical values.

Bias (Volts)	Dark Current Density ($Amps/cm^2$)	Experimental Noise Current Density ($A / cmHz^{1/2}$)	Theoretical Noise Current Density ($A / cmHz^{1/2}$)
0.75	5.0×10^{-5}	1.6×10^{-12}	1.0×10^{-12}
1.00	1.0×10^{-4}	2.8×10^{-12}	2.0×10^{-12}
1.25	1.8×10^{-4}	4.2×10^{-12}	3.6×10^{-12}
1.50	2.8×10^{-4}	6.0×10^{-12}	5.7×10^{-12}
1.75	3.9×10^{-4}	7.5×10^{-12}	7.8×10^{-12}
2.00	4.5×10^{-4}	9.0×10^{-12}	8.9×10^{-12}
2.25	5.5×10^{-4}	1.0×10^{-11}	1.1×10^{-11}
2.50	6.0×10^{-4}	1.1×10^{-11}	1.2×10^{-11}
2.75	7.0×10^{-4}	1.2×10^{-11}	1.4×10^{-11}
3.00	8.0×10^{-4}	1.3×10^{-11}	1.6×10^{-11}
3.25	9.0×10^{-4}	1.4×10^{-11}	1.8×10^{-11}
3.50	1.0×10^{-3}	1.5×10^{-11}	2.0×10^{-11}
3.75	1.2×10^{-3}	1.6×10^{-11}	2.4×10^{-11}
4.00	1.4×10^{-3}	1.7×10^{-11}	2.8×10^{-11}

Experimental and theoretical noise current density as a function of bias voltage for Thibaudeau's $GaAs/Al_{0.263}Ga_{0.737}As$ QWIP sample 'A'.



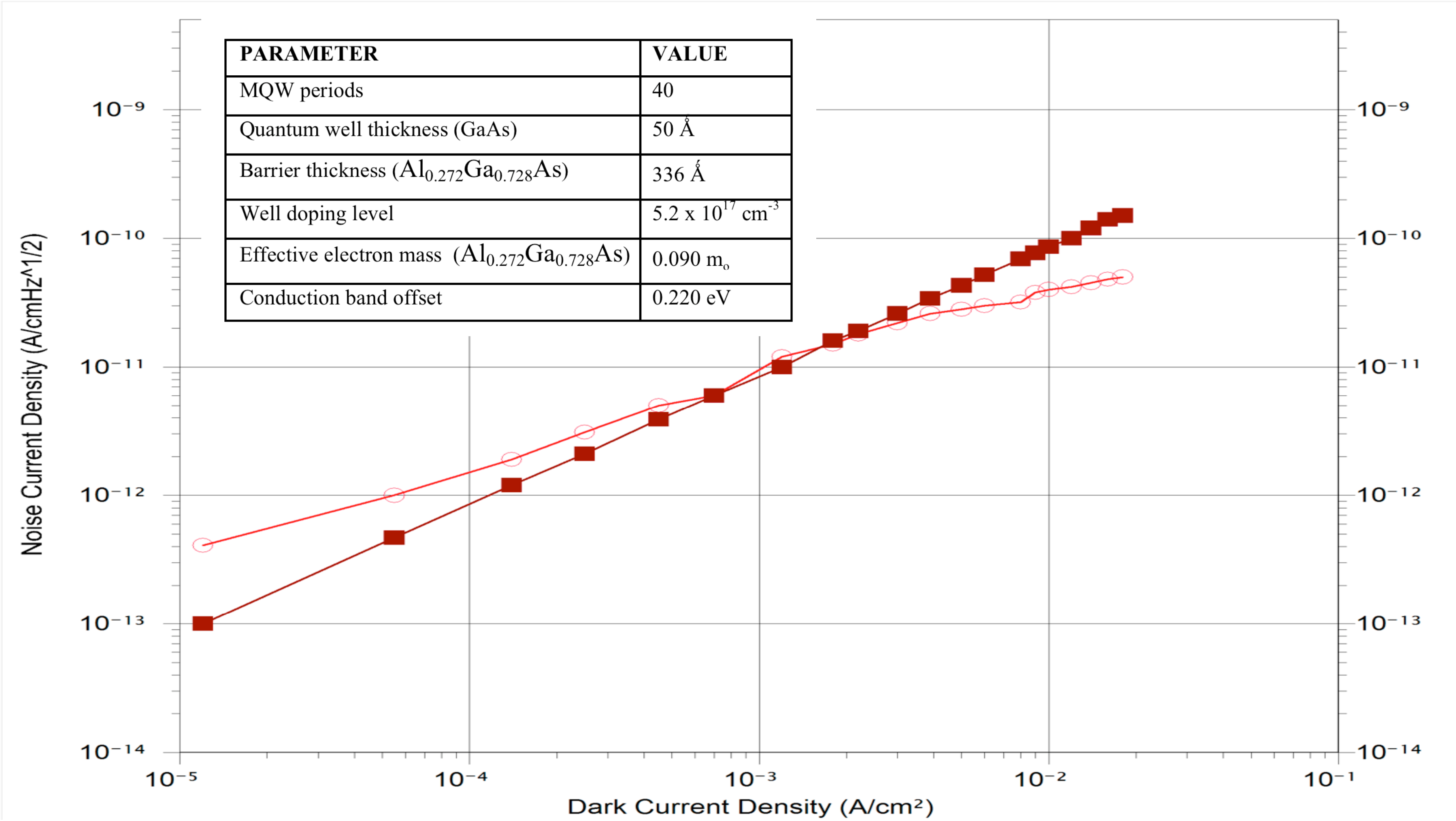
PARAMETER	VALUE
MQW periods	40
Quantum well thickness (GaAs)	52 Å
Barrier thickness ($Al_{0.263}Ga_{0.737}As$)	342 Å
Well doping level	4.7 x 10^{17} cm^{-3}
Effective electron mass ($Al_{0.263}Ga_{0.737}As$)	0.089 m_o
Conduction band offset	0.227 eV

L. Thibaudeau, P. Bois, and J. Y. Duboz, "A Self – Consistent Model for Quantum Well Infrared Photodetectors," J. Appl. Phys., **79** No.1, pp. 446 (1996).

Experimental and theoretical noise current density vs. dark current density for the case of Thibaudeau's GaAs/$Al_{0.263}Ga_{0.737}As$ QWIP sample 'A'. Squares represent theoretical values.

Bias (Volts)	Dark Current Density (Amps/cm^2)	Experimental Noise Current Density (A / cmHz$^{1/2}$)	Theoretical Noise Current Density (A / cmHz$^{1/2}$)
0.2	1.2×10^{-5}	4.1×10^{-13}	1.0×10^{-13}
0.4	5.5×10^{-5}	1.0×10^{-12}	4.7×10^{-13}
0.6	1.4×10^{-4}	1.9×10^{-12}	1.2×10^{-12}
0.8	2.5×10^{-4}	3.1×10^{-12}	2.1×10^{-12}
1.0	4.5×10^{-4}	5.0×10^{-12}	3.9×10^{-12}
1.2	7.0×10^{-4}	6.0×10^{-12}	6.0×10^{-12}
1.4	1.2×10^{-3}	1.2×10^{-11}	1.0×10^{-11}
1.6	1.8×10^{-3}	1.5×10^{-11}	1.6×10^{-11}
1.8	2.2×10^{-3}	1.8×10^{-11}	1.9×10^{-11}
2.0	3.0×10^{-3}	2.2×10^{-11}	2.6×10^{-11}
2.2	3.9×10^{-3}	2.6×10^{-11}	3.4×10^{-11}
2.4	5.0×10^{-3}	2.8×10^{-11}	4.3×10^{-11}
2.6	6.0×10^{-3}	3.0×10^{-11}	5.2×10^{-11}
2.8	8.0×10^{-3}	3.2×10^{-11}	6.9×10^{-11}
3.0	9.0×10^{-3}	3.8×10^{-11}	7.7×10^{-11}
3.2	1.0×10^{-2}	4.0×10^{-11}	8.6×10^{-11}
3.4	1.2×10^{-2}	4.2×10^{-11}	1.0×10^{-10}
3.6	1.4×10^{-2}	4.5×10^{-11}	1.2×10^{-10}
3.8	1.6×10^{-2}	4.8×10^{-11}	1.4×10^{-10}
4.0	1.8×10^{-2}	5.0×10^{-11}	1.5×10^{-10}

Experimental and theoretical noise current density as a function of bias voltage for Thibaudeau's GaAs/$Al_{0.272}Ga_{0.728}As$ QWIP sample 'B'.



Experimental and theoretical noise current density vs. dark current density for the case of Thibaudeau's GaAs/$Al_{0.272}Ga_{0.728}$As QWIP sample 'B'. Squares represent theoretical values.

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These important results prove that the quantum 1/f theory is capable of predicting low frequency noise spectra for any given QWIP device (where thermionic emission is the dominant mechanism) and can be used as valuable tool for device improvement and noise minimization for optimal signal detection.

[1] A. Truong, "Quantum 1/f Noise in Infrared Photodetectors and Scanning Tunneling Microscopes", Univ. Of Mo Ph.D. Thesis, St. Louis MO (2010).
[2] J.Jiang, "Infrared focal plane array based on GaInAs/InP QWIPs", Northwestern U. Ph.D. Thesis, Evanston IL (2003).
[3] C. Jelen, S. Slivken, V. Guzman, M. Razeghi, and G. Brown, "InGaAlAs/InP Quantum Well Infrared Photodetectors for 8 – 20 µm Wavelengths," IEEE J. Quantum Electronics, **34** No.10, pp. 1873 (1998).
[4] L. Thibaudeau, P. Bois, and J. Y. Duboz, "A Self – Consistent Model for Quantum Well Infrared Photodetectors," J. Appl. Phys., **79** No.1, pp. 446 (1996).

3. Conclusions

- From Eqs. (3.18), (3.12), and (3.8) we see that the quantum 1/f formulas correctly describe the 1/f noise in the dark current of QWIPs, both in magnitude and in its dependence on bias
- The photo-generation process does not have quantum 1/f noise
- New analytical formulas allow for the first time optimization of the QWIPs with respect to parameters such as choice of the semiconductors used, i.e., the effective mass of the carriers, band and subband parameters, well geometry, etc.
  - Quantum well intersubband photodetectors (QWIPs) can be extended in principle from infrared into the THz region
  - **For other devices and systems, see e.g., A. van der Ziel**: "Unified Presentation of 1/f Noise in Electronic Devices; Fundamental 1/f Noise Sources", **Proc. of the IEEE, 76, 233-258 (1988); F. Sthal, M. Devel, S. Ghosh, J. Imbaud, G. Cibiel and R. Bourquin:**

"Volume dependence in Handel's model of quartz crystal resonator noise" IEEE Transactions on UFFC, 60, 1971-1977, (2013).

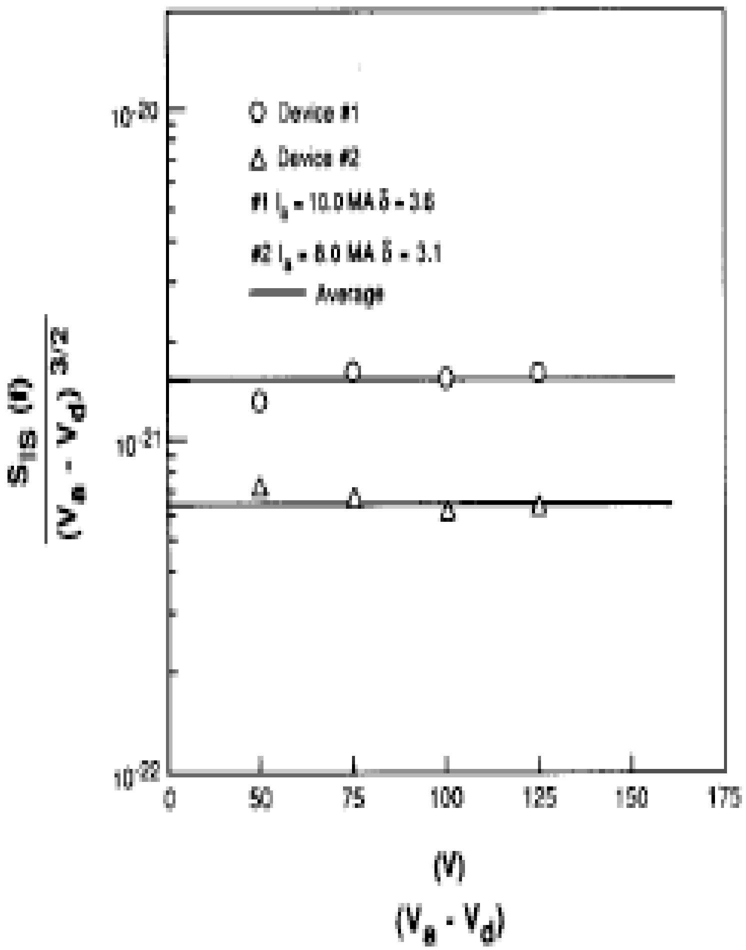

Fig. 2. $S_{I_a}(10)/(V_a - V_d)^{3/2}$ for EFP60 as a function at $(V_a - V_d)$. Cathode $1/f$ noise removed by feedback. I_a and δ were constant.

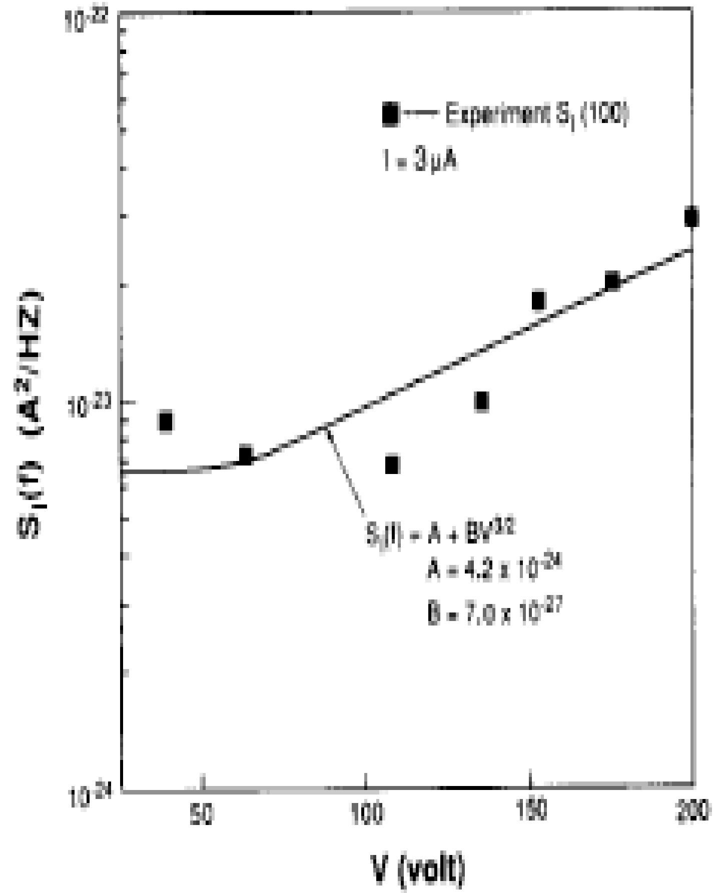

Fig. 3. $S_{I_a}(10)$ versus V_a for vacuum photodiode of 931A photomultiplier. All dynodes connected to d_1 (diode configuration). Full-drawn curve: $A + BV_a^{3/2}$; symbols represent data. A represents work function $1/f$ noise (adjustable parameter) and B is taken from (64) (theory) with $I_a = 3\mu A$, $d_{ca} = 0.50$ cm, $n = 2$.

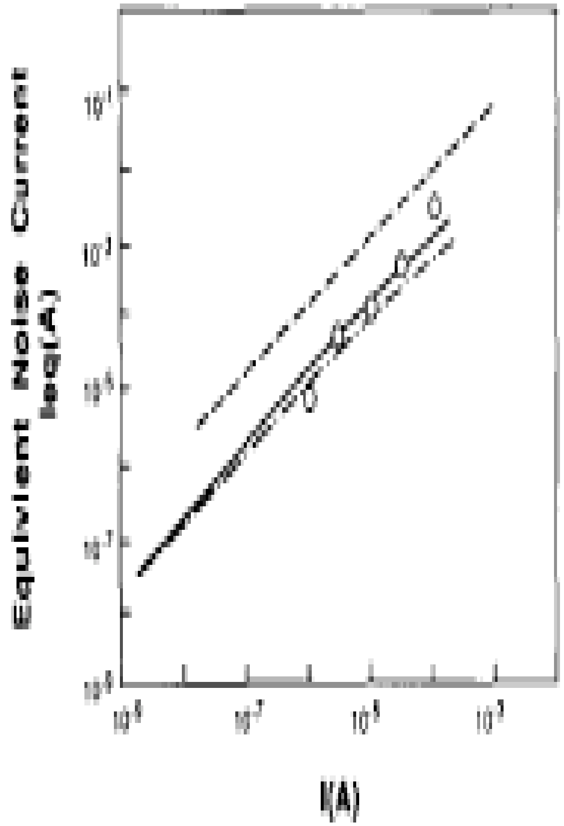

Fig. 4. $I_{eq}(f)$ versus I for Pt-n-Si in Schottky-barrier diodes at 20 Hz. Upper broken line: diffusion model. Lower broken line: ballistic model. Full line: TE model involving image effect; circles: Hsu's data.

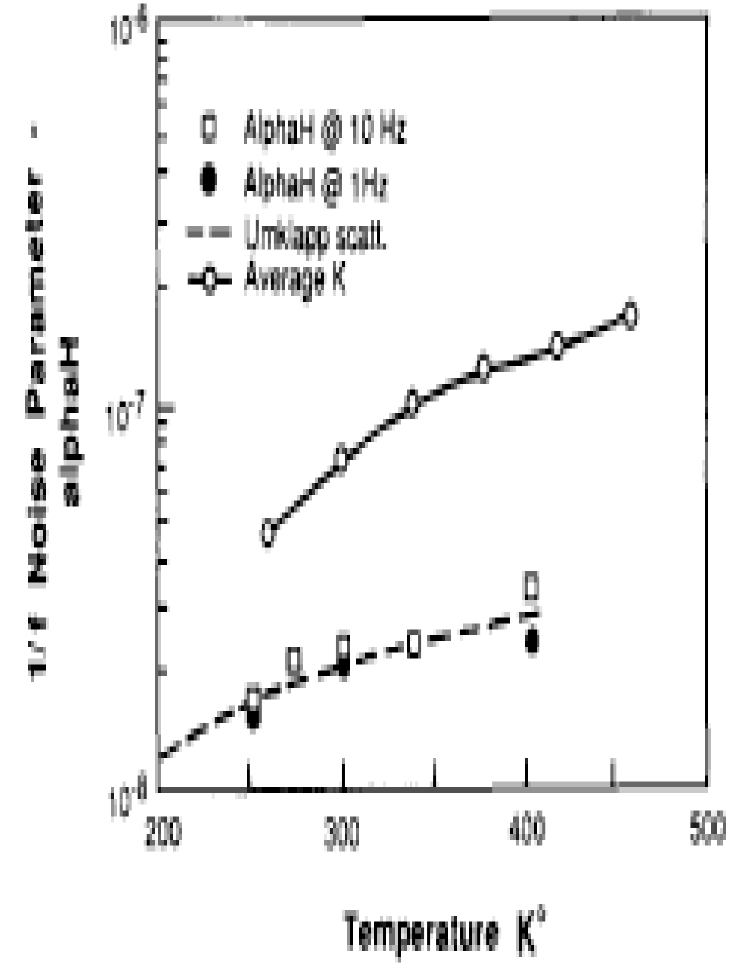

Fig. 5. α_{Hn} versus T for Si n-channel JFET. Comparison of the measured α_H with (67b). Also shown is $\alpha_H(T)$ for the inter-valley process.

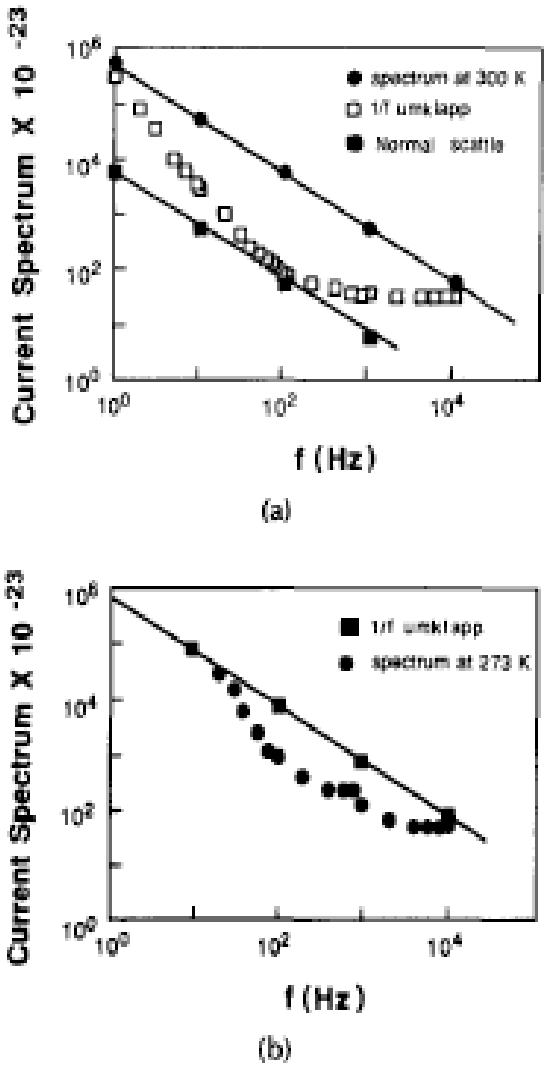

Fig. 6. $S_I(f)_{exp}$ versus frequency in Si p-channel JFETs. (a) Theoretical spectrum for $\alpha_{Hp} = 4.2 \times 10^{-7}$ (see (67b)). (b) Theoretical spectrum for $\alpha_{Hp} = 10^{-8}$ (phonon collision model). In cases (a) and (b) the h.f. thermal noise was added.

Duh [25] measured α_{Hp} in p-channel MOSFETs of 7.5- and 17.5-μm length, respectively; they were test devices obtained from Harris Semiconductor at Melbourne (FL) and had $\alpha_{Hp} = (3\text{–}9) \times 10^{-7}$. A typical plot of α_{Hp} versus $-V_d$ is shown in Fig. 7. We note that α_{Hd} is practically independent

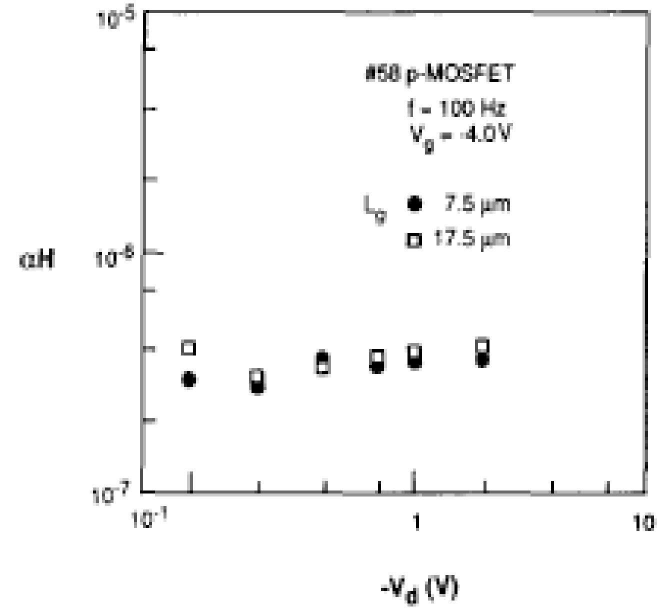

Fig. 7. α_{Hp} versus V_d for p-channel MOSFETs at channel length $L = 7.5$ and $17.5\ \mu$m. $V_g = -4.0$ V, $f = 100$ Hz.

of $(-V_d)$ and independent of the device length L, as expected for Umklapp $1/f$ noise. It is therefore quite likely that Duh actually observed Umklapp $1/f$ noise. The direct proof would be to measure the temperature dependence of α_{Hp}. Unfortunately, the devices were lost before tests could be made; a search for similar low-noise devices is in progress.

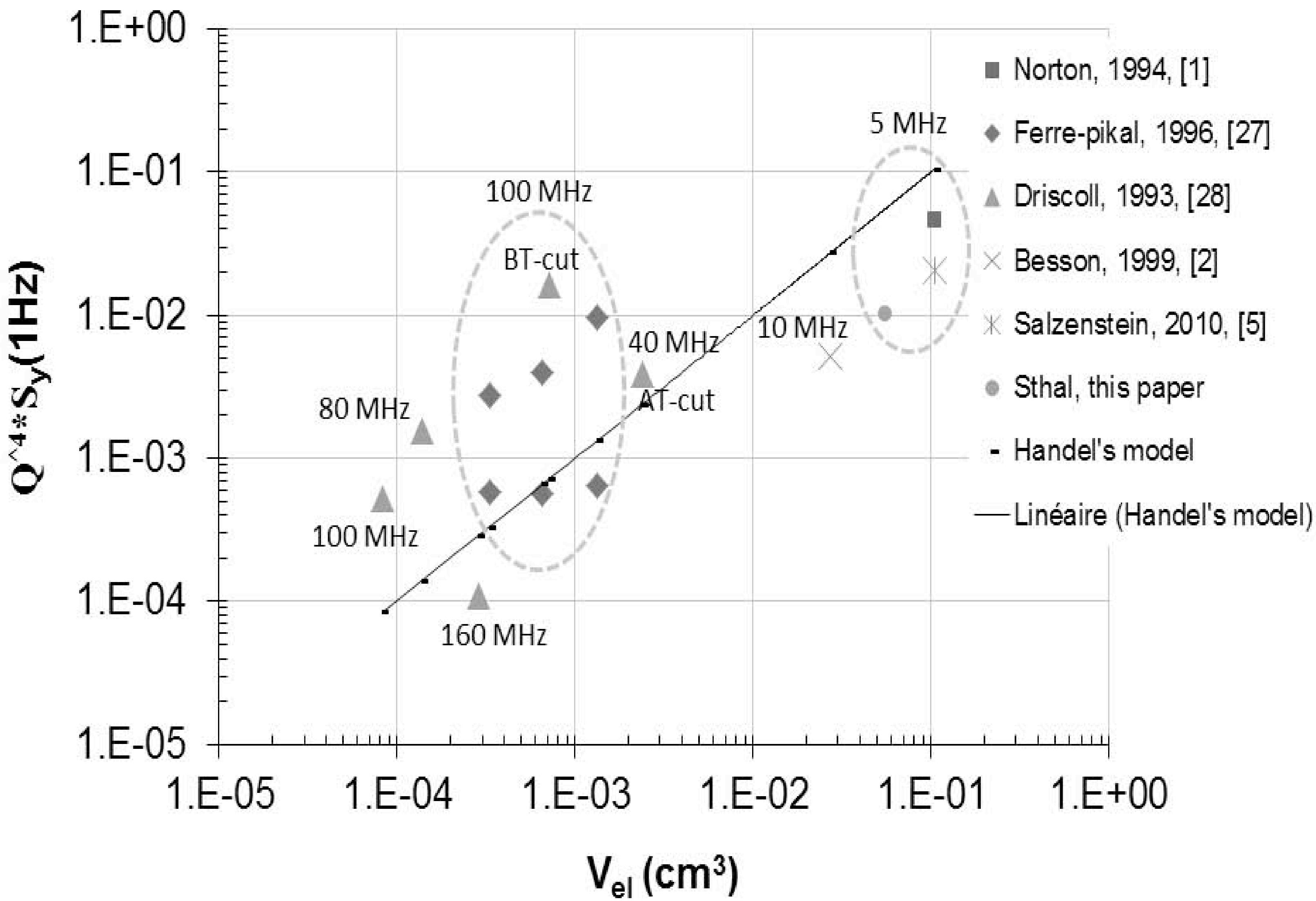

Fig. 3. $Q^4 \cdot S_y$ as a function of the volume between the electrodes of the resonator, showing experimental points from various authors plus a straight line for Handel's prediction with $\beta = 1\ \mathrm{cm}^{-3}$.

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