Decoherence and Conventional Quantum 1/f Noise

Peter H. Handel
Department of Physics & Astronomy and
Center for Nanoscience, University of Missouri-St. Louis
Saint Louis, MO 63121, USA
handel@umsl.edu

Abstract— **The conventional Quantum 1/f Effect is present in any cross section or process rate involving charged particles or current carriers. The present paper shows how bremsstrahlung and decoherence at all frequencies yield probability density fluctuations at all frequencies in the outgoing beam, that are observed as 1/f noise and frequency fluctuations, or phase noise, in materials, devices and systems.**

Keywords—1/f Noise, Quantum Mechanics, Quantum 1/f Noise, Decoherence, Quantum 1/f Effect, Low-frequency fluctuations, Electronic Noise, Phase Noise.

I. Introduction

1/f noise has been a problem for engineers since the early 1920's, when they first learned to amplify electric signals. Much effort has been directed toward controlling it by trial and error in vacuum tubes, carbon resistors and microphones, contacts, etc. Later, it slowed the application of transistors and other semiconductor devices, where again, its general nature remained unknown, in spite of generally unsuccessful attempts to explain it as a fortuitous superposition of lorentzian spectra caused by slow surface states, or by migrating impurities, or by other diffusion processes. 1/f spectra are ubiquitous.

Since 1962-1966, I understood the universality of 1/f noise as the result of the nonlinear reaction of currents back on themselves, as first described by my magneto-hydrodynamic plasma turbulence theory of 1/f noise, developed [1], [2] by generalization of Heisenberg's first theory of hydrodynamic turbulence. Our turbulence theory was well received [3], but could only explain the spectrum, not also the intensity of low frequency noise. The latter was to be determined by the instabilities of laminary flow, which were causing instabilities and homogeneous, isotropic, turbulence.

My ensuing 12-year hunt for new types of instabilities yielded some, but none with zero threshold, as would have been required in order to explain 1/f noise. This led to my sustained attempts to quantize the turbulence theory. The result was the quantum theory of 1/f noise, pure QED, which included the conventional and coherent quantum 1/f effect.

The conventional Q1/fE was introduced as a fundamental fluctuation of all currents of charged particles j, of all scattering cross sections σ_s, recombination cross sections σ_r, tunneling rates or transition rates Γ of any kind, in short of all physical cross sections σ and process rates Γ, given by the universal formula $S(f) = 2\alpha A/fN$ (conventional quantum 1/f equation [4]-[21]) for small devices, and $S(f) = 2\alpha/\pi fN$ (coherent quantum 1/f equation [22], [23], [18]-[21]) for large devices. These two Q1/fE forms can be combined into a single general formula [22], [23], [18]-[21], as we show below. Here S(f) is the spectral density of fractional fluctuations in current, $\delta j/j$, in energy and momentum components which interfere with each other, causing conventional quantum 1/f noise, i.e., quantum fluctuations of σ with a 1/f spectral density. The quantum expectation $<\sigma>$ of physical σ gives the usual definition of the cross section. Here $\alpha = e^2/c\hbar = 1/137$ is Sommerfeld's fine structure constant. $A = 2(\Delta\mathbf{v}/c)^2/3\pi$ is essentially the square of the vector velocity change $\Delta\mathbf{v}$ of the scattered particles in the scattering process whose fluctuations we are considering, in units of c. Finally, N is the number of particles used to define the notion of current j, of cross section σ or of process rate Γ. The Q1/fE is quasi-macroscopic because of the 1/f dependence.

Another manifestation, or aspect, of the same Q1/fE phenomenon is the uncertainty of the mass shell of a charged particle due to the infinite range of the Coulomb interaction [24]-[28]. This requires a new picture [24]-[28] in which the long range part of the interaction is included in the unperturbed hamiltonian, which leads to the existence of the interaction representation, to finite matrix elements, to a branch-point singularity in the propagator, to a more complex free particle notion, to a coherent state of the field associated with the free particle, and as we prove below, to the presence of macroscopic coherent quantum 1/f fluctuations of all charged currents (coherent quantum 1/f effect). The quantum information theory approach yields [29] one of the alternative derivations.

The physical origin of quantum 1/f noise is easy to understand. Consider for example Coulomb scattering of current carriers, e.g., electrons on a center of force. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by DeBroglie waves of a frequency corresponding to their energy. However, part of the wave function of the scattered electrons has lost energy in the scattering process, due to the amplitude for emission of bremsstrahlung. Therefore, some components of the outgoing DeBroglie waves are shifted to slightly lower frequencies. Decoherence will randomize the phases of these components. When we calculate the probability density in the scattered beam, we obtain also cross terms, linear both in the part scattered with and without bremsstrahlung. These cross terms contain these random phases and oscillate with the same frequency as the frequency of the emitted bremsstrahlung photons.

The corresponding current density fluctuations present in the current, emerging from some scattering or other interaction process, are obtained by multiplying the probability density fluctuations by the velocity of the scattered current carriers. Finally, these current fluctuations present in the scattered beam will be noticed at the detector as low frequency current fluctuations, and will be interpreted as fundamental cross section fluctuations in the scattering cross section of the arbitrary force center. While incoming carriers may have been Poisson distributed, the scattered beam will exhibit super-Poissonian statistics, or bunching, due to this new effect which we may call quantum 1/f effect. This quantum 1/f effect is thus a many-body or collective effect, at least a two-particle effect, best described through the two-particle wave function and two-particle correlation function.

II. Derivation of the conventional quantum 1/f effect

This derivation includes three simple steps:

1. First, I refer to the usual QED derivation of scattering with bremsstrahlung, resulting in the well-known 1/f dependence of the squared matrix elements' absolute value for scattering with energy loss hf through the emission of a photon of frequency f, resulting in a perceived "infrared catastrophe." The catastrophe is eliminated by including all real and virtual multi-phonon processes leading to the energy loss hf, but a $1/f^{1-\alpha A}$ finally replaces the 1/f dependence, with $\alpha A \ll 1$.

2. Second, in a process known as decoherence, the phases attached to these matrix elements, to the final bremsstrahlung states, or scattering channels, are randomized due to interactions with the rest of the world. This simplifies the derivation. The absolute value $\sim f^{-1/2}$ of the matrix elements is well known.

3. Third, I calculate the fluctuations in the resulting mixture of states, characterized by the presence of random phases in each channel that displays an energy loss hf, and obtain a universal 1/f spectrum of extreme low frequency (infrared-divergent) quantum fluctuations in the scattered current. I call this e.l.f fluctuating scattered current per unit incoming flux "the Physical scattering Cross Section," or "Physical Process Rate" in general. They still include the genuine "infrared," i.e., ultra-low-frequency (macroscopic, or mesoscopic) quantum fluctuations with a low-frequency limit given just by the reciprocal time of the 1/f noise measurement. Their expectation value would be the usual notion of cross section or process rate. They both do not exist in the limit for αA going to zero, or even in general, due to $\alpha A \ll 1$, since we bump into the finite age of the universe. Indeed, for a physical observable limited by fluctuations with a C/f spectrum, the Allan variance is 2C ln2, independent of the measurement duration, provided it is long enough to average out thermal other stationary noise. The conventional Quantum 1/f Effect (Q1/fE) [8]-[10],[12]-[13],[20]-[23], is present in any cross section or process rate involving charged particles or current carriers.

I therefore start, e.g., with the expression of the Heisenberg representation mixed state $|S\rangle$ of N identical bosons of mass M emerging at an angle θ from some scattering process with various undetermined bremsstrahlung energy losses reflected in their one-particle waves $\varphi_i(\xi_i)$

$$\begin{aligned} |S\rangle &= (N!)^{-1/2}\Pi_i \int d^3\xi_i \varphi_i(\xi_i)\psi^+(\xi_i)|0\rangle \\ &= \Pi_i \int d^3\xi_i \varphi_i(\xi_i)|S^0\rangle \end{aligned} \qquad (1)$$

where $\psi^+(\xi_i)$ is the field operator creating a boson with position vector ξ_i and $|0\rangle$ is the vacuum state, while $|S^0\rangle$ is the state with N bosons of position vectors ξ_i with i = 1.....N. All products and sums in this paper run from 1 to N, unless otherwise stated.

To calculate the particle density autocorrelation function in the outgoing scattered wave, we need the expectation value of the operator

$$O(x_1,x_2) = \psi^+(x_1)\psi^+(x_2)\psi(x_2)\psi(x_1) \qquad (2)$$

known as the operator of the pair correlation. Using the commutation properties of the boson field operators, we first calculate the matrix element

$$\begin{aligned} &N!\langle S^0|O|S^0\rangle \\ &= \sum\nolimits'_{\mu\nu}\sum\nolimits'_{mn}\delta(\eta_\nu - x_1)\delta(\eta_\mu - x_2)\delta(\xi_n - x_1)\delta(\xi_m - x_2)\sum\nolimits_{ij}\prod\nolimits'_{ij}\delta(\eta_j - \xi_i) \end{aligned} \qquad (3)$$

Here the prime excludes $\mu=\nu$ and m=n in the summations and excludes i=m, i=n, $j=\mu$ and $j=\nu$ in the product. The summation $\Sigma_{(i,j)}$ runs over all permutations of the remaining N-2 values of i and j. On the basis of this result we now calculate the complete matrix element

$$\begin{aligned} \langle S|O|S\rangle &= [1/N(N-1)]\sum\nolimits'_{\mu\nu}\sum\nolimits'_{mn}\int d^3\eta_\mu \int d^3\eta_\nu \int d^3\xi_m \int d^3\xi_n \\ &\varphi^*_\mu(\eta_\mu)\varphi^*_\nu(\eta_\nu)\varphi_m(\xi_m)\varphi_n(\xi_n)\delta(\eta_\nu - x_1)\delta(\eta_\mu - x_2)\delta(\xi_n - x_1)\delta(\xi_m - x_2) \\ &= [1/N(N-1)]\sum\nolimits'_{\mu\nu}\sum\nolimits'_{mn}\varphi^*_\mu(x_2)\varphi^*_\nu(x_1)\varphi_m(x_1)\varphi_n(x_2) \end{aligned} \qquad (4)$$

The one-particle states are spherical waves emerging from the scattering center located at x=0.

$$\varphi(x) = (C/x)e^{iKx}\left[1 + \sum\nolimits_{k1} b(k,1)e^{-iqx}a^+k,1\right] \qquad (5)$$

Here C is an amplitude factor, K the boson wave vector magnitude, b(k,l) the bremsstrahlung amplitude for photons of wave vector k and polarization l, while $a^+_{k,l}$ is the corresponding photon creation operator, allowing the emitted photon state to be created from the vacuum, if this is inserted into Eq. (1). Note that the same results are obtained if a random phase factor $\exp(i\gamma_{kl})$ is replacing the creation operators. The random phase factor is introduced by the decoherence process following the scattering with bremsstrahlung. The momentum magnitude loss

$$\hbar q = Mck/K \equiv 2\pi Mf/K \tag{6}$$

is necessary for energy conservation in the Bremsstrahlung process. Substituting our φ(x) into Eq. (4), we obtain

$$\langle S|O|S\rangle = |C/x|^4 \left\{ N(N-1) + 2(N-1)\sum_{k,1} |b(k,1)|^2 \left[1+\cos q(x_1-x_2)\right]\right\} \tag{7}$$

where we neglected a small term of higher order in b(**k**,l). To perform the angular part of the summation in Eq. (7), we calculate the current expectation value of φ(**x**), and compare it to the well known cross section without and with bremsstrahlung

$$j = \left(\hbar K/Mx^2\right)\left[1+\sum_{kl}|b(k,1)|^2\right] = j_0\left[1+\alpha A\int df/f\right] \tag{8}$$

where the quantum fluctuations have disappeared, $\alpha = e^2/hc$ is the fine structure constant, $\alpha A = (2\alpha/3\pi)(\Delta \mathbf{v}/c)^2$ is the fractional bremsstrahlung rate coefficient, also known in QED as the infrared exponent, and where the 1/f dependence of the bremsstrahlung part displays the well-known apparent infrared catastrophe, (discussed above with $1/f^{1-\alpha A}$ in place of 1/f) i.e., the emission of a logarithmically divergent number of photons in the low frequency limit. Here $\Delta\mathbf{v}$ is the velocity change $\hbar(\mathbf{K}-\mathbf{K}_0)/M$ of the scattered boson, and $f = ck/2\pi$ the photon frequency. Eq. (7) thus gives

$$\langle S|O|S\rangle = |C/x|^4 \left\{ N(N-1) + 2(N-1)\alpha A\int\left[1+\cos q(x_1-x_2)\right]df/f\right\} \tag{9}$$

which is the pair correlation function, or density autocorrelation function along the scattered beam with df/f = dq/q. The spatial distribution fluctuations along the scattered beam will also be observed as fluctuations in time at the detector, at any frequency f. According to the Wiener-Khintchine theorem, we obtain the spectral density of fractional scattered particle density ρ, or current

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/fN \tag{10}$$

where N is the number of particles or current carriers used to define the current j whose fluctuations we are studying. Quantum 1/f noise is thus a fundamental 1/N effect. The exact value of the exponent of f in Eq. (10) can be determined by including the contributions from all real and virtual multiphoton processes of any order (infrared radiative corrections), and turns out to be αA-1, rather than -1, which is important only philosophically, since αA<<1. The spectral integral is thus convergent at f = 0. This ends our 3 steps of the derivation. The conventional quantum 1/f effect derived here has many applications, allowing for the ultra-low noise and phase noise optimization of most high-tech materials, devices, and systems.[30]-[32]

III. Conclusions

The decoherence step was not emphasized sufficiently in past derivations of the conventional quantum 1/f effect in physical cross sections and process rates. This might have led some scientists to feel that the conventional quantum 1/f effect, which is irreversible, includes an increase of entropy violates the principles of quantum mechanics. Indeed, if we would start with a pure state of the electrons, the electron scattering force center or potential, and of the electromagnetic field's ground state, we should end up with a pure state in the end. Both the initial and final entropy would be zero, and in this ideal Gedankenexperiment conventional quantum 1/f noise would be impossible.

Nobody has formulated a criticism in that way, but I imagine that this may have contributed to the uneasy "gut feeling" of many. The decoherence step removes this difficulty. It represents in essence the inescapable interaction of the electrons, and of our system in general, with the rest of the world. This is also why Schrödinger's cat is never found in a superposition of states, dead and alive.

We are left, however, with a possible question about the possibility of incomplete decoherence, and with an associated possible quantum 1/f noise reduction factor. This reminds us that the measured Q1/fE in the β-radioactive decay rate is usually an order of magnitude below the simplistic theoretical estimate. [33-35]

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