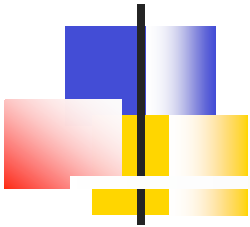
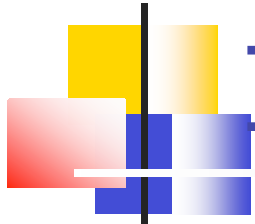


FUNDAMENTAL $1/f$ PIEZOELECTRIC TRANSDUCER SIZE FLUCTUATIONS



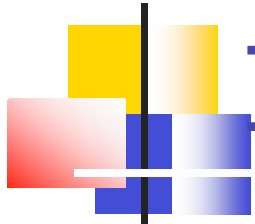
Peter H. Handel, Amanda M. Truong and Kobra Nasiri
Avanaki

Department of Physics and Astronomy and Center for
Nanoscience,
Univ. of Missouri, St. Louis, MO 63121, USA



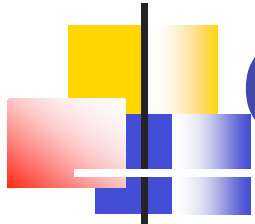
Introduction

- The feedback loop of the Scanning Tunneling Microscope (STM) maintains a constant current throughout the circuit.
- A piezoelectric crystal that controls the height of the tip is a key component in this feedback loop.
- The fluctuations in the tunneling current and voltage fluctuations in the piezoelectric crystal will result in deviations in tip-sample separation distance.



Introduction

- The fluctuations in tip-sample separation distance are related to the fluctuations in the phonon emission rate, G .
- The phonon emission within the piezoelectric quartz crystal can be caused by scattering events with higher frequency phonons, or with defects or impurities occurring in the crystal.
- The phonon emission rate G is equal to $1/\tau$, where τ is the relaxation rate of the quartz.

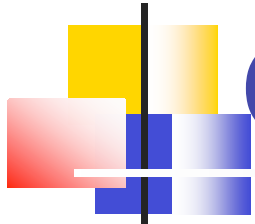


Quantum 1/f Formulation

- At high temperatures, where $\omega\tau \ll 1$, scattering processes lead to velocity changes, which take the general form:

$$v - v_0 = \frac{CT \langle \gamma_{eff}^2 \rangle}{2\rho v_0} \frac{\Omega_0^2 \tau^2}{1 + \Omega_0^2 \tau^2}$$

where C is the specific heat, T is the temperature, γ is the Grüneisen constant v_0 is the speed of sound in the crystal, Ω_0 is the resonator frequency of the crystal, ρ is the crystal density, and τ is the mean phonon relaxation time



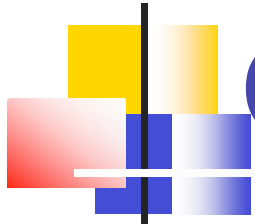
Quantum 1/f Formulation

- To simplify, let $\Delta E = CT \langle \gamma^2 \rangle = \text{constant}$ and also substitute the low frequency Young modulus $E_0 = \rho v_0^2 = \text{constant}$, and $x = \Omega_0^2 \tau^2$
- This yields:

$$\delta E = \Delta E \frac{\delta x}{(1+x)^2}$$

- The variation in x can be expressed in terms of the variation in τ

$$\delta x = \delta (\Omega_0^2 \tau^2) = 2\Omega_0^2 \tau^2 \frac{\delta \tau}{\tau} = 2x \frac{\delta \tau}{\tau}$$



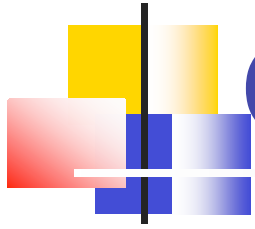
Quantum 1/f Formulation

- This allows for the Young's modulus to be expressed in terms of the variance in the relaxation time $\delta\tau$

$$\delta E = \Delta E \frac{2x}{(1+x)^2} \frac{\delta\tau}{\tau}$$

or equivalently

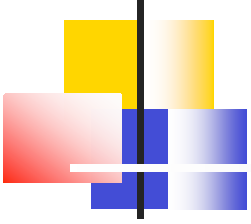
$$\left\langle (\delta E)^2 \right\rangle_f = 4(\Delta E)^2 \frac{x^2}{(1+x)^4} \left\langle \left(\frac{\delta\tau}{\tau} \right)^2 \right\rangle_f$$



Quantum 1/f Formulation

- The fluctuation in the Young modulus is proportional to fluctuations in the spatial displacement X of the piezoelectric crystal
- The force, F_{app} , exerted on the crystal to cause a displacement X can be written

$$X = \frac{F_{app}}{A} \cdot \frac{X_0}{E}$$



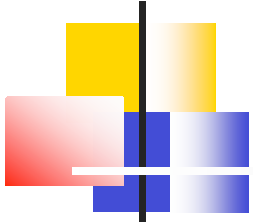
Quantum 1/f Formulation

- According to the quantum $1/f$ theory, the spectral density of fractional fluctuations, $S_{\delta X/X}$, is related to both the spectral density of total fluctuations, $S_{\delta X}$, and the variance

$$\left\langle (\delta X)^2 \right\rangle_f = S_{\delta X}(f) = X^2 \cdot S_{\frac{\delta X}{X}}(f)$$

or

$$S_{\frac{\delta X}{X}} = S_{\frac{\delta E}{E}} = \frac{1}{E^2} S_{\delta E}$$

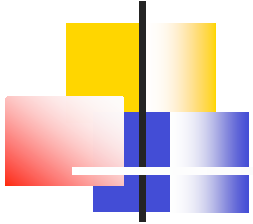


Quantum 1/f Formulation

The quartz quality factor, Q , is related to the phonon wave attenuation in the crystal, α' . The relation is defined by

$$\alpha' = \frac{\Omega_0}{2Qv_0} = \frac{CT\Omega_0 \langle \gamma^2 \rangle_{eff}}{2\rho v_0^3} \frac{\Omega_0 \tau}{1 + \Omega_0^2 \tau_0^2}$$

where C is the specific heat, T is the temperature, γ is the Grüneisen constant, v_0 is the speed of sound in the crystal, Ω_0 is the resonator frequency of the crystal, ρ is the crystal density, and τ is the mean phonon relaxation time.

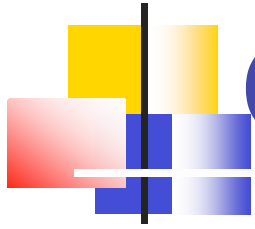


Quantum 1/f Formulation

Simplifying as before, in terms of the Young's modulus, we obtain

$$\frac{1}{Q} = \frac{\Delta E}{E_0} \frac{\Omega_0 \tau}{1 + \Omega_0^2 \tau^2} = \frac{\Delta E}{E_0} \frac{\sqrt{x}}{1 + x}$$

A. M. Truong and P. H. Handel, "1/f performance limits of scanning tunneling microscopes," in *Noise and Fluctuations* Proc 20th Intl. Conf. on Noise and Fluctuations, M. Macucci and G. Basso Eds., ICNF'09, ISBN 978-07354-0665-0; ISSN 0094-243X, pp. 69-72.



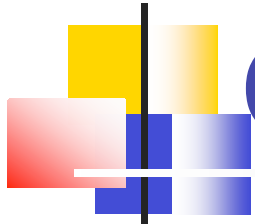
Quantum 1/f Formulation

- Finally, we obtain

$$S_{\frac{\delta X}{X}} = 4 \left(\frac{\Delta E}{E} \right)^2 \frac{x^2}{(1+x)^4} S_{\frac{\delta \tau}{\tau}}$$

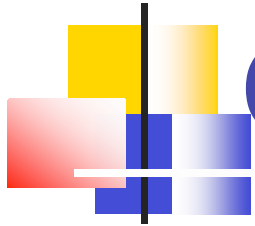
which can also be expressed in terms of the quartz Q-factor

$$S_{\frac{\delta X}{X}} \approx \frac{4}{Q^4} S_{\frac{\delta \Gamma}{\Gamma}}$$



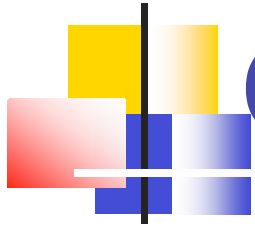
Quantum $1/f$ Formulation

- The rate Γ of phonon interactions which remove phonons from the main quartz resonator mode is modulated by the quantum $1/f$ effect
- Thereby exhibiting observable quantum fluctuations, while the expectation value of these interactions remains constant



Quantum 1/f Formulation

- When a phonon is removed from the main resonator mode, the time derivative $d\mathbf{P}/dt$ of the quartz dipole moment vector $\mathbf{P}$, is suddenly affected
- This modulation produces radiation
- The amount of energy radiated away is constant, per unit frequency interval



Quantum 1/f Formulation

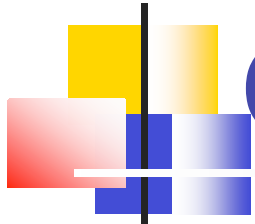
- The energy W of the interacting resonator mode $\langle \omega \rangle$ can be expressed

$$W = n\hbar \langle \omega \rangle = \frac{m}{Ne^2} \chi^2 (\dot{P})^2$$

where

$$\Delta \dot{P} = \frac{1}{2n} \left(\sqrt{\frac{nN\hbar \langle \omega \rangle}{m}} \right) e = \frac{e}{2} \sqrt{\frac{N\hbar \langle \omega \rangle}{nm}}$$

A. M. Truong and P. H. Handel, "1/f performance limits of scanning tunneling microscopes," in *Noise and Fluctuations* Proc 20th Intl. Conf. on Noise and Fluctuations, M. Macucci and G. Basso Eds., ICNF'09, ISBN 978-07354-0665-0; ISSN 0094-243X, pp. 69-72.

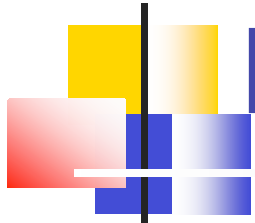


Quantum 1/f Formulation

- Now the spectral density of fractional spatial fluctuations can be written [4] in terms of the spectral density of fractional fluctuations of the phonon emission rate Γ , by multiplication with $4/Q^4$

$$S_{\frac{\delta X}{X}} \approx \frac{4}{Q^4} \left[\frac{N\alpha\hbar\langle\omega_{eff}\rangle}{3\pi nMc^2 f} \right] = \frac{C}{f}$$

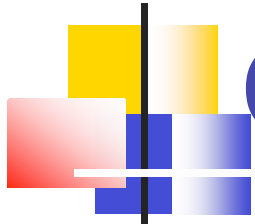
P.H. Handel, A.G. Tournier, "Nanoscale Engineering for Reducing Phase Noise in Electronic Devices," Proceedings of the IEEE, vol. 93, No. 10, pp. 1784-1814 (2005).



Results

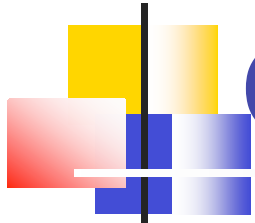
- In the case that the coherence length of the phonons is smaller than the crystal size, we expect several incoherent noise contributions from various regions of the crystal
- For a 1 cm quartz,

$$\frac{\delta X}{X} = \sqrt{\frac{3.6 \times 10^{-17}}{\text{Hz}}} = \frac{6 \times 10^{-9}}{\sqrt{\text{Hz}}}$$



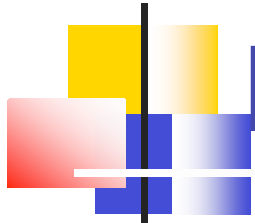
Conclusion

- As a consequence of this result, the quantum $1/f$ fluctuations in this spatial direction are $0.6 \text{ \AA}$ per square root of Hertz.
- This means, for instance, that the piezoelectric crystal in a STM contributes 1000 times more error in tip sample separation distance compared with the quantum $1/f$ fluctuations occurring in a stationary needle without the feedback loop installed



Conclusion

- In general, we get the Allan variance: $\langle (dX)^2 \rangle = 2C \ln 2$
- Therefore, the quantum $1/f$ fluctuation in the size of the piezoelectric crystal controlling the STM tip are not negligible
- These fluctuations represent a fundamental limitation in any piezoelectric transducer
- The formulas derived here can be used for the optimization of electro-acoustic transducers of any kind, in particular sonic focal plane arrays for acoustic imaging purposes



References

- [1] J.J. Gagnepain, J. Uebersfeld, G. Goujon, P.H. Handel: "Relation between $1/f$ Noise and Q-factor in Quartz Resonators at Room and Low Temperatures," 35th Annual Symposium on Frequency Control, Philadelphia, pp. 476-483 (1981).

- [2] P.H. Handel: "Noise, Low frequency," Wiley Encyclopedia of Electrical and Electronics Engineering, vol. 14, pp. 428-449, John Wiley & Sons, Inc., John G. Webster, Editor, (1999).

- [3] P.H. Handel, A.G. Tournier, "Nanoscale Engineering for Reducing Phase Noise in Electronic Devices," Proceedings of the IEEE, vol. 93, No. 10, pp. 1784-1814 (2005).

- [4] A. M. Truong and P. H. Handel, " $1/f$ performance limits of scanning tunneling microscopes," in *Noise and Fluctuations* Proc 20th Intl. Conf. on Noise and Fluctuations, M. Macucci and G. Basso Eds., ICNF'09, ISBN 978-07354-0665-0; ISSN 0094-243X, pp. 69-72.