FUNDAMENTAL 1/F PIEZOELECTRIC TRANSDUCER SIZE FLUCTUATIONS

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Introduction

- The feedback loop of the Scanning Tunneling Microscope (STM) maintains a constant current throughout the circuit.
- A piezoelectric crystal that controls the height of the tip is a key component in this feedback loop.
- The fluctuations in the tunneling current and voltage fluctuations in the piezoelectric crystal will result in deviations in tip-sample separation distance.

Introduction

- The fluctuations in tip-sample separation distance are related to the fluctuations in the phonon emission rate, G.
- The phonon emission within the piezoelectric quartz crystal can be caused by scattering events with higher frequency phonons, or with defects or impurities occurring in the crystal.
- The phonon emission rate G is equal to $1/\tau$, where τ is the relaxation rate of the quartz.

Quantum 1/f Formulation

- At high temperatures, where ωτ << 1, scattering processes lead to velocity changes, which take the general form:

$$v - v_0 = \frac{CT\left\langle \gamma_{eff}^2 \right\rangle}{2\rho v_0} \frac{\Omega_0^2 \tau^2}{1 + \Omega_0^2 \tau^2}$$

where C is the specific heat, T is the temperature, γ is the Grüneisen constant v_0 is the speed of sound in the crystal, Ω_0 is the resonator frequency of the crystal, ρ is the crystal density, and τ is the mean phonon relaxation time

Quantum 1/f Formulation

- To simplify, let ΔE = CT $<\gamma^2>$ = constant and also substitute the low frequency Young modulus $E_0 = \rho\ v_0^2$ = constant, and $x = \Omega_0^2\,\tau^2$
- This yields:

$$\delta E = \Delta E \frac{\delta x}{(1+x)^2}$$

- The variation in x can be expressed in terms of the variation in τ

$$\delta x = \delta\left(\Omega_0^2 \tau^2\right) = 2\Omega_0^2 \tau^2 \frac{\delta\tau}{\tau} = 2x\frac{\delta\tau}{\tau}$$

Quantum 1/f Formulation

- This allows for the Young's modulus to be expressed in terms of the variance in the relaxation time $\delta\tau$

$$\delta E = \Delta E \frac{2x}{(1+x)^2} \frac{\delta\tau}{\tau}$$

or equivalently

$$\left\langle (\delta E)^2 \right\rangle_f = 4(\Delta E)^2 \frac{x^2}{(1+x)^4} \left\langle \left(\frac{\delta\tau}{\tau} \right)^2 \right\rangle_f$$

Quantum 1/f Formulation

- The fluctuation in the Young modulus is proportional to fluctuations in the spatial displacement *X* of the piezoelectric crystal
- The force, F_{app}, exerted on the crystal to cause a displacement *X* can be written

$$X = \frac{F_{app}}{A} \cdot \frac{X_0}{E}$$

Quantum 1/f Formulation

- According to the quantum *1/f* theory, the spectral density of fractional fluctuations, $S_{\delta X/X}$, is related to both the spectral density of total fluctuations, $S_{\delta X}$, and the variance

$$\left\langle (\delta X)^2 \right\rangle_f = S_{\delta X}(f) = X^2 \cdot S_{\frac{\delta X}{X}}(f)$$

or

$$S_{\frac{\delta X}{X}} = S_{\frac{\delta E}{E}} = \frac{1}{E^2} S_{\delta E}$$

Quantum 1/f Formulation

The quartz quality factor, Q , is related to the phonon wave attenuation in the crystal, α′. The relation is defined by

$$\alpha' = \frac{\Omega_0}{2Qv_0} = \frac{CT\Omega_0 \left\langle \gamma^2 \right\rangle_{eff}}{2\rho v_0^3} \frac{\Omega_0 \tau}{1+\Omega_0^2 \tau_0^2}$$

where C is the specific heat, T is the temperature, γ is the Grüneisen constant, v_0 is the speed of sound in the crystal, Ω_0 is the resonator frequency of the crystal, ρ is the crystal density, and τ is the mean phonon relaxation time.

Quantum 1/f Formulation

Simplifying as before, in terms of the Young's modulus, we obtain

$$\frac{1}{Q} = \frac{\Delta E}{E_0} \frac{\Omega_0 \tau}{1+\Omega_0^2 \tau^2} = \frac{\Delta E}{E_0} \frac{\sqrt{x}}{1+x}$$

A. M. Truong and P. H. Handel, "1/f performance limits of scanning tunneling microscopes," in *Noise and Fluctuations* Proc 20th Intl. Conf. on Noise and Fluctuations, M. Macucci and G. Basso Eds., ICNF'09, ISBN 978-07354-0665-0; ISSN 0094-243X, pp. 69-72.

Quantum 1/f Formulation

- Finally, we obtain

$$S_{\frac{\delta X}{X}} = 4\left(\frac{\Delta E}{E}\right)^2 \frac{x^2}{(1+x)^4} S_{\frac{\delta\tau}{\tau}}$$

which can also be expressed in terms of the quartz Q-factor

$$S_{\frac{\delta X}{X}} \approx \frac{4}{Q^4} S_{\frac{\delta\Gamma}{\Gamma}}$$

Quantum 1/f Formulation

- The rate Γ of phonon interactions which remove phonons from the main quartz resonator mode is modulated by the quantum *1/f* effect
- Thereby exhibiting observable quantum fluctuations, while the expectation value of these interactions remains constant

Quantum 1/f Formulation

- When a phonon is removed from the main resonator mode, the time derivative d***P***/dt of the quartz dipole moment vector ***P***, is suddenly affected
- This modulation produces radiation
- The amount of energy radiated away is constant, per unit frequency interval

Quantum 1/f Formulation

- The energy *W* of the interacting resonator mode <ω> can be expressed

$$W = n\hbar\langle\omega\rangle = \frac{m}{Ne^2}\chi^2\left(\dot{P}\right)^2$$

where

$$\Delta\dot{P} = \frac{1}{2n}\left(\sqrt{\frac{nN\hbar\langle\omega\rangle}{m}}\right)e = \frac{e}{2}\sqrt{\frac{N\hbar\langle\omega\rangle}{nm}}$$

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Quantum 1/f Formulation

- Now the spectral density of fractional spatial fluctuations can be written [4] in terms of the spectral density of fractional fluctuations of the phonon emission rate Γ, by multiplication with $4/Q^4$

$$S_{\frac{\delta X}{X}} \approx \frac{4}{Q^4}\left[\frac{N\alpha\hbar\langle\omega_{eff}\rangle}{3\pi n M c^2 f}\right] = \frac{C}{f}$$

P.H. Handel, A.G. Tournier, "Nanoscale Engineering for Reducing Phase Noise in Electronic Devices," Proceedings of the IEEE, vol. 93, No. 10, pp. 1784-1814 (2005).

Results

- In the case that the coherence length of the phonons is smaller than the crystal size, we expect several incoherent noise contributions from various regions of the crystal
- For a 1 cm quartz,

$$\frac{\delta X}{X} = \sqrt{\frac{3.6x10^{-17}}{Hz}} = \frac{6x10^{-9}}{\sqrt{Hz}}$$

Conclusion

- As a consequence of this result, the quantum 1/f fluctuations in this spatial direction are 0.6 Å per square root of Hertz.
- This means, for instance, that the piezoelectric crystal in a STM contributes 1000 times more error in tip sample separation distance compared with the quantum 1/f fluctuations occurring in a stationary needle without the feedback loop installed

Conclusion

- In general, we get the Allan variance: $<(dX)^2> = 2C\ln 2$
- Therefore, the quantum 1/f fluctuation in the size of the piezoelectric crystal controlling the STM tip are not negligible
- These fluctuations represent a fundamental limitation in any piezoelectric transducer
- The formulas derived here can be used for the optimization of electro-acoustic transducers of any kind, in particular sonic focal plane arrays for acoustic imaging purposes

References

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[4] A. M. Truong and P. H. Handel, "1/f performance limits of scanning tunneling microscopes," in *Noise and Fluctuations* Proc 20th Intl. Conf. on Noise and Fluctuations, M. Macucci and G. Basso Eds., ICNF'09, ISBN 978-07354-0665-0; ISSN 0094-243X, pp. 69-72.