Noise Limitations of FET-Based Biochemical Sensors

Peter H. Handel[a] and Amanda M. Truong[a]

[a]*Department of Physics and Astronomy and Center for Nanoscience*
University of Missouri, Saint Louis, 1 University Boulevard, Saint Louis, MO 63121, USA

Abstract. The gate surface of an FET can be painted with antibodies for certain biological agents, causing it to change its surface potential by δV_G and therefore also the channel resistance by $\delta R_{Ch} = \kappa\delta m$ when agents of mass δm are adsorbed. Here κ is the effectivity of the antibody paint. The resulting channel current change competes with quantum 1/f noise generated in the channel. This competition determines the sensitivity. By choosing the right chemical activation deposit on the gate surface, a chemical sensor is obtained. In the present paper for the first time a complete, reliable, analytical formula is provided for the flicker floor of the FET-based biological and chemical sensors, allowing for their simpler optimization.

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QUANTUM THEORY OF 1/F NOISE

The 1/f resistance fluctuations δR in a biased semiconductor yield voltage fluctuations. They are caused by conventional and coherent quantum 1/f effects (Q1/fE), given by the general quantum 1/f formula [1]-[5], shown below in Eq. (3). It contains the coherence parameter $s = 2N'r_0$ where N' is the number of current carriers per unit length in the direction of current flow, and $r_0 = e^2/mc^2 = 2.84.10^{-13}$ cm is the classical radius of the electron.

The Coherent Q1/fE arises in a current of charged particles, from the definition of the electron as a bare particle plus a coherent state of the electromagnetic field. The expression for the power spectral density of fractional fluctuations given by the coherent quantum 1/f theory is

$$S^{coh}_{\frac{\delta R}{R}}(f) = \frac{2\alpha}{\pi f N} \tag{1}$$

The Conventional Q1/fE is a fundamental quantum fluctuation of physical cross sections σ and process rates Γ. The Conventional Q1/fE is caused by the bremsstrahlung energy and momentum losses of charged particles, when they are scattered or accelerated in any way. The spectral density is given by

$$S^{conv}_{\frac{\delta R}{R}}(f) = \frac{4\alpha}{3\pi f N}\left(\frac{\Delta v}{c}\right)^2 \tag{2}$$

Both components of quantum 1/f noise are combined in the general quantum 1/f formula

$$S_{\frac{\delta R}{R}} = \frac{1}{1+s} S_{\frac{\delta R}{R}}^{conv}(f) + \frac{s}{1+s} S_{\frac{\delta R}{R}}^{coh}(f) \qquad (3)$$

where the coherence parameter, $s = 2N'r_0$, indicates to what extent the energy of the drift motion of the electrons adds up coherently or conventionally over the cross section of the current, in the Hamiltonian. The spectral density is related to the fractional fluctuations in each region using the following relation

$$S_{\delta R} = R^2 \cdot S_{\frac{\delta R}{R}}(f) \qquad (4)$$

For V_{DS} constant, the spectral densities of current and resistance fluctuations are related by

$$\frac{S_{\delta R}}{R^2} = \frac{S_{\delta I}}{I^2} \qquad (5)$$

Quantum 1/f Noise in the FET Channel

The element of resistance dR along the current-carrying channel can be written $dR = dy/(q\mu_n N')$, where μ_n is the electron mobility [6]. The quantum 1/f noise in the element of resistance dR is given from Eq. (3) by the basic quantum 1/f formula for mobility fluctuations, $(dR)^{-2}\langle(\delta dR)^2\rangle_f = \alpha_H/fdN$, where $dN=N'dy=ntdy$ is the number of carriers in the element of length dy along the channel of thickness t and concentration n. The 1/N dependence of Eqs. (1)-(3) is used to define an effective Hooge parameter $\alpha_H = \alpha_{eff}$ shown below Eq. (8). Substituting, we obtain $f\langle(\delta dR)^2\rangle_f = [dRdy/q\mu_n N']a_H/dN$. Multiplying on both sides with the channel current, $I_{ch} = -dV/dR$, and integrating, yields

$$fI_{ch}\langle(\delta R)^2\rangle_f = -\int_{Vs}^{Vd}[dV/q\mu_n N'^2] \qquad (6)$$

Introducing Fermi statistics, for a $GaN/Al_xGa_{1-x}N$ doped n-channel HFET the number of carriers per unit length N' of the 2-dimensional channel is written in the form $N'(y) = ZD_{eff}V_{th}\log\{1 + \exp[(V_G^*-V(y))/V_{th}]\}$, where $D_{eff} = qm/\pi\hbar$ is q times the effective density of states per unit area. Multiplying Eq. (6) by I/f, this yields the spectral density of V_{DS} fluctuations in terms of device width Z, channel current I, and thermal voltage $V_{Th}=kT/q$, with $X=1+\exp[(V_G^*-V)/V_{th}]$

$$\langle(\delta V_{DS})^2\rangle_f = \frac{|I|}{fZ^2D_{eff}qV_{th}} \int_{X_0}^{X_1} (\alpha_H/\mu_n)[dX/(X-1)\ln^2 X] \qquad (7)$$

Here X_0 and X_1 are the values of X at the source (V=0) and at the drain ($V=V_{DS}$), while $\alpha_H = \alpha_H(y)$ is the y-dependent quantum 1/f coefficient.

In this case of FETs and HFETs of much larger width $w \gg L_{DS} > t$, the kinetic energy E_k of average motion with drift velocity v_d per unit length in the direction of L_{DS} is still given by $Nmv_d^2/2$, but the magnetic energy E_m per unit length in the direction of L_{DS} is roughly proportional with the first power of w only, instead of w^2, and can be approximated by $E_m = \pi[\ln(w/2L_{DS})]L_{DS}[nevS/c]^2/w$. This indicates decoherence along the device width w. This yields a coherence ratio of $s \equiv E_m/E_k \approx \pi n r_o t L_{DS}\ln(w/2L_{DS})$, which shows that only an effective width $w = w_{eff}$ about equal to L_{DS} should be used in the calculation of the coherence parameter s in this special case; larger widths are subject to de-coherence. This favors lower, mainly conventional, quantum 1/f noise in these devices, in spite of the large values of w. It also explains for the first time why the huge widths are possible with impunity, i.e., without causing the much larger coherent quantum 1/f noise to affect the device.

In general, part of the channel may be sub-threshold (the part closer to the drain), starting at $V(y) = V_{sat} = V_G^*$. The use of Fermi statistics becomes unavoidable, as is shown in Eq. (4). However, although Eq. (4) includes the field dependence of α and μ, it fails to reflect the physical situation correctly, because it neglects the impact ionization effect in the subthreshold section of the channel length. This effect is roughly included below in a first approximation, along with the variation of AlGaN polarization with y, by dividing the channel along its length into coherent and conventional parts.

Using the fact that the velocity of the carriers shows strong saturation in the subthreshold section, to include the impact ionization effect, we return to the variable V in Eq. (4), we split up the integral at $V(y) = V_G^* - V_{th}$, and write the second, sub-threshold, part of the integral separately, with the number of carriers per unit channel length defined as $N' = I_{ch}/qv_s$, where v_s is the saturation velocity,

$$
\begin{aligned}
fI_{Ch} < (\delta R)^2 >_f &= \int_0^{V_{sat}} \frac{\alpha_{eff} dV}{q\mu N'^2} + \int_{V_{sat}}^{V_{DS}} \frac{\alpha_{conv} q v_s^2 dV}{q\mu I_{Ch}^2} \\
&\approx \frac{V_{DS}}{\mu I_{Ch}} \{ \frac{\alpha_{eff} I_{ch}}{qZ^2 D_{eff}^2 V_{th} V_{DS}} + \frac{\alpha_{conv} q v_s^2}{I_{ch}} (1 - \frac{V_G^*}{V_{DS}} + \frac{V_{th}}{V_{DS}}) \} \qquad (8) \\
&= \kappa^2 fI_{Ch} \left\langle (\delta m)^2 \right\rangle_f = \kappa^2 I_{Ch} C;
\end{aligned}
$$

where $!_{eff} = \frac{1}{1+s} !_{conv} + \frac{s}{1+s} !_{coh}$, and C is the coefficient in $<(\delta m)^2>_f = C/f$, defined by Eq. (8). The second term in curly brackets must be omitted when negative. Note, however, that the first integral was calculated here neglecting the variation of α_{eff}, and the second integral was performed neglecting the variation of v_s.

With this quantum 1/f definition of C in Eq. (8), *the ultimate two-sample (Allan) variance obtainable is* $<(dm)^2> = 2C\ln 2$. This limits the achievable minimum detectable concentration of the biological agent.

DISCUSSION

Our calculation of the fundamental 1/f noise in the channel is applicable both to HFET-like and FET-like biochemical sensors. It shows that the ultimate achievable detection limit is proportional to the biological or chemical effectivity κ of the substance (e.g., an antibody to a certain bacterium) that was deposited on the gate, as defined by Eq. (8). From a physical point of view, the direct action of the biological agent is on the surface potential V_G of the FET-like structure. The change in V_G, in turn, acts on the carrier density and energy level structure of the 2-D electron gas in the channel, thus causing a change in the channel current I and resistance R at constant applied bias V_{DS}. If the change δV_G in surface potential is small, we can introduce a coefficient k as the change δV_G per unit mass δm attracted by the antibody deposit on the gate. We obtain for constant V_{DS},

$$\delta R/R = -\delta I/I = -g_m \delta V_G/I = -g_m k \delta m/I = \kappa \delta m/R, \text{ and } \kappa = -g_m k R^2/V_{DS}, \tag{9}$$

where g_m is the trans-conductance of the FET. The two constants κ and k characterizing the effectivity of an antibody or chemically sensitive gate deposit, are thus related by the family of I/V characteristics and the implied value of g_m. Multiplying Eq. (8) with $I/(V_{DS})$, we finally obtain for the smallest detectable amount δm the (Allan) two-sample variance

$$\begin{aligned} (\delta m_{rms})^2 &= 2C \ln 2 = \frac{2f \ln 2}{\kappa^2} < (\delta R)^2 >_f = \\ &\approx \frac{2R^2 \ln 2}{\kappa^2 \mu V_{DS}} \{ \frac{\alpha_{eff} I_{ch}}{qZ^2 D_{eff}^2 V_{th} V_{DS}} + \frac{\alpha_{conv} q v_s^2}{I_{ch}} (1 - \frac{V_G^*}{V_{DS}} + \frac{V_{th}}{V_{DS}}) \Theta() \}; \end{aligned} \tag{10}$$

Here $\Theta()$ is the step function of the preceding round bracket, zero for negative values and 1 for positive values of the bracket.

The above expression provides an analytical expression for the achievable flicker floor of biological and chemical detection, that can't be reduced by longer time averaging. Eq. (10) can also be used to optimize industrial FET-based biochemical sensors and detectors.

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