Quantum 1/f Noise, a New Aspect of Quantum Physics in Hi-Tech Devices, Sensors, Nanostructures and Systems

Peter H. Handel* Amanda M. Truong*, Thomas F. George* and Hadis Morkoç+

*Department of Physics and Astronomy and Center for Nanoscience
University of Missouri-Saint Louis, #1 University Blvd. Saint Louis, MO 63121
+Virginia Commonwealth University, Richmond, VA, USA

Abstract. An investigation of electronic 1/f noise in ultrasmall devices and systems is presented, focused on nanoscale engineering of electronic devices for low phase noise. The investigation is based on the quantum 1/f formulas and the results are compared with the experimental evidence. The role of phonons is studied in particular. In addition, the s-parameters for QED, piezoelectric and quantum gravidynamic coherent and conventional quantum 1/f effects are presented and discussed. Practical applications are also presented.

Keywords: Bio-sensors, Chemical sensors, Quantum 1/f noise, Phase noise, FET, HFET.

1. INTRODUCTION

The Quantum 1/f Effect (Q1/fE) [1]-[5] is a fundamental fluctuation of all currents of charged particles j, of all scattering cross sections σ_s, recombination cross sections σ_r, tunneling rates or transition rates of any kind Γ, in short of all physical cross sections σ and process rates Γ, given by the universal formula

$$S(f) = 2\alpha A/fN \tag{1.1}$$

(conventional quantum 1/f equation [4]-[5]) for small devices, and

$$S(f) = 2\alpha/\pi fN \tag{1.2}$$

(coherent quantum 1/f equation [2]-[5]) for large devices. These two Q1/fE forms can be combined into a single general formula [2]-[5], as indicated below in Eq. (1.3). Here S(f) is the spectral density of fractional fluctuations in current, $\delta j/j$, in scattering or recombination cross section $\delta\sigma/\sigma$, or in any other process rate $\delta\Gamma/\Gamma$. $\alpha=e^2/\hbar c=1/137$ is Sommerfeld's fine structure constant, a magic number of our world depending only on Planck's constant $\hbar$, the charge of the electron e and the speed of light in vacuum c. $A=2(\Delta\mathbf{v}/c)^2/3\pi$ is essentially the square of the vector velocity change $\Delta\mathbf{v}$ of the scattered particles in the scattering process whose fluctuations we are considering, in units of c. Finally, N is the number of particles used to define the notion of current j, of cross section σ or of process rate Γ.

Due to the presence of a bremsstrahlung emission amplitude in scattering, the scattered current per unit incoming flux, also called by us "the physical scattering cross section σ" will contain several energy and momentum components. These interfere with each other, causing conventional quantum 1/f noise, i.e., quantum fluctuations of σ with a 1/f spectral density. The quantum expectation $<\sigma>$ of σ gives the usual definition of the cross section. The quantum 1/f fluctuations are macroscopic at low frequencies because of the 1/f dependence, and because they are slow, thus quite different from other quantum fluctuations, a new aspect. As seen from Eqs. (1.1) and (1.2) they contain Planck's constant in the denominator. Therefore, they will be important at low frequencies, as opposed to other quantum fluctuations.

Another manifestation, or aspect, of the same Q1/fE phenomenon is the uncertainty of the mass shell of a charged particle due to the infinite range of the Coulomb interaction [2]-[4]. This requires a new picture [2]-[4] in which the long range part of the interaction is included in the unperturbed hamiltonian. This, in turn, leads for the first time to the existence of the interaction representation, and to finite matrix elements. It also leads to a branch-point singularity in the propagator, to a more complex free particle notion, to a coherent state of the field associated with the free particle. As has been shown, however, it also leads to the presence of the macroscopic coherent quantum 1/f fluctuations of all charged currents, expressed by Eq. (1.2), i.e., the coherent quantum 1/f effect.

We expect the total observed spectral density of the mobility fluctuations to be approximated by a relation of the form

$$(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN] \qquad (1.3)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side. In this equation $\alpha A=2\alpha(\Delta\mathbf{v}/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum 1/f effect. The coherence parameter s is given by the "coherence ratio"

$$s = E_m/E_k = 2ne^2S/mc^2[1/4 + \ln(R/r)] \approx 2e^2N'/mc^2 = 2N'r_o = 5.68\cdot10^{-13}\text{cm}\cdot N', \quad (1.4)$$

where $N' = nS$ is the number of carriers per unit length of the sample, $r_o= e^2/mc^2 = 2.84\cdot10^{-13}$ cm is the classical radius of the electron, and the expression in rectangular brackets has been approximated by unity in the last form. Note that the coherence ratio s introduced here equals the unity for the critical value $N' = N" = 2\cdot10^{12}$/cm., e.g. for a cross section $S = 2\cdot10^{-4}$ cm^2 of the sample when $n = 10^{16}$. For small samples with $N'<<N"$ only the first term survives, while for $N'>N"$ the second term in Eq. (1.3) is dominant.

2. CALCULATION OF THE s-PARAMETER FOR THE CASE OF HFETs AND FETs OF LARGE WIDTH

An important special case should be mentioned. Eq. (1.3) was derived for the case of a cylindrical sample with circular cross section of radius r, shaped like a straight wire. Modern FETs and HFETs try to increase the denominator N in Eqs. (1.1)-(1.3) without markedly increasing the coherence parameter s, by choosing an extreme case of a rectangular cross section for the portion between source and drain, and even more extreme for the channel, which is the region under the gate. Indeed, the circular cross section is replaced by a rectangle with a very large width w of several hundreds of microns and less than a micron thick, the channel being even thinner. The channel is reduced to a 2-dimensional quantum sheet, which is a quantum well of size (thickness) t in the 3rd dimension, towards the gate. Most important, the "length" L_{DS} of the source-to-drain distance is only of the order of 1micron, while the gate length is just about a quarter micron or less. In this case the kinetic energy E_k per unit length in the direction of L_{DS} is not affected in Eq. (1.3), but the magnetic energy E_m per unit length in the direction of L_{DS} is roughly proportional with the first power of the width w only, instead of w^2, and can be approximated by

$$E_m = \pi[\ln(w/2L_{DS})]L_{DS}[nevS/c]^2/w. \tag{2.1}$$

This indicates decoherence along the device width w. For $w \gg L_{DS} > t$, this yields a coherence ratio

$$s \approx \pi n r_o t L_{DS} \ln(w/2L_{DS}) \text{ for } w \gg L_{DS} > t, \tag{2.2}$$

which shows that only an effective width $w = w_{eff}$ about equal to L_{DS} should be used in the calculation of the coherence parameter s in this special case; larger widths are subject to de-coherence, i.e., their magnetic energy no longer increases proportional to w^2, or N'^2. This favors lower, mainly conventional, quantum 1/f noise in these devices, in spite of the large values of w, in agreement with the practical results, which anticipated the quantum 1/f result simply by trial and error, unaware of the subtleties of the transition from coherent to conventional Q1/f noise in this case, and in general. It also shows that fragmentation of the extremely large widths w does not reduce 1/f noise in this special case.

Note: To derive the rough approximation given in Eq. (2.1), one writes the expression of the coherent ($\propto n^2$) contribution of the currents in the short, but very wide channel to the magnetic energy of the circuit in the form

$$E_m = \int[\mathbf{j}(\mathbf{x})\cdot\mathbf{j}(\mathbf{x}')/2c^2|\mathbf{x}-\mathbf{x}'|]d^3x d^3x' = e^2 v_d^2 \int[n(\mathbf{x})\cdot n(\mathbf{x}')/2c^2|\mathbf{x}-\mathbf{x}'|]d^3x d^3x'. \tag{2.3}$$

The integral coincides with the electrostatic energy of an equivalent constant charge density numerically equal to e/c^2 in CGS, or $e\mu_o/4\pi\varepsilon_0$ in SI, uniformly distributed over the volume wtL_{DS} of the channel. This energy, in turn, is calculated as the corresponding volume integral over the squared equivalent electric field, using L_{DS} as a lower cut-off. The equivalent electric field is $netw/cr(r^2+w^2/4)^{1/2}$ at a distance r in the median plane of a segment of length w oriented along the width w of the channel. A more elaborate calculation does not simply approximate the field with its value in this median plane, but yields similar results in the end.

3. QUANTUM 1/F PHASE NOISE IN OSCILLATORS

3.1 FET and HFET - Based Amplifier Noise Figure F when the 1/f Noise Power Spectra $S_{\delta R/R}$ for the Channel and $S_{\delta RG/RG}$ for the Gate are Known

The connection between HFET channel resistance fluctuation noise and the noise figure F of the corresponding amplifier is derived here, in order to show how the various noise sources present in the channel and in the gate can be integrated.

The Noise Figure F of an amplifier stage is defined as

F = [Output noise power $<\Delta V^2>_f$]/[Output noise power of the noiseless amplifier or device].

If, for instance, the amplifier stage (e.g., an FET or HFET), amplifies the thermal noise of a conductance g_s connected between the input ports, with a current I_{Ch} present in the channel, the noise figure is

$$F = \{4kT_o g_s + (4kT_o g_i + I_G^2 S_{\delta RG/RG}) + (g_s+g_i)^2[(4kT_o/R_{Ch} + I_{Ch}^2 S_{\delta R/R})/|Y_{tr}|^2]\}/4kT_o g_s$$
$$= 1 + [g_i + I_G^2 S_{\delta RG/RG}/4kT_o]/g_s + (g_s+g_i)^2\{[4kT_o/R_{Ch} + I_{Ch}^2 S_{\delta R/R}]/|Y_{tr}|^2\}/4kT_o g_s \quad (3.1)$$

Here $S_{\delta R/R}$ and $S_{\delta RG/RG}$ are the (quantum 1/f and GR -type) spectral densities of fractional resistance fluctuations in the channel resistance R_{Ch} and gate insulation conductance g_i respectively, while $|Y_{tr}|$ is the transadmittance, also known as transconductance (g_m) at lower frequencies. This result derived below for F assumes that the noise source $S_{\delta RG/RG}$ in the gate insulation conductance g_i and the noise source $S_{\delta R/R}$ in the channel resistance R_{Ch} are uncorrelated. This is not always rigorously true, particularly at high frequencies, at which, however, the base-band 1/f noise is negligible; while phase noise matters. The phase noise is the one that persists with importance at high frequencies, more than amplitude noise, in most applications.

The minimal noise figure F_{min} is a good measure of the noise of the amplifier stage at intermediate values of g_s, R_{no} is a good noise measure for large values of g_s, and $g_{no}= g_n+ R_n g_i^2$ is a good noise measure for small values of g_s. From Eqs. (1) and (5), $R_{no}= R_n/(1-2R_n g_i) \approx R_n$ is the source resistance necessary to double the output power of the (short-circuited-input) amplifier stage. Also, $g_{no}/(1-2R_n g_i) \approx g_{no}$ is the source conductance needed in order to double the output of an amplifier, starting from $g_s=0$. Note that R_n and g_n are defined by Eqs. (8) and (9) below. *Finally, F turns out to be practically the same for common source, common gate and common drain circuits*. The $\approx$ sign assumes $g_i <<1/2R_n$, i.e., that there is good gate insulation. The minimal noise figure

$$F_{min} = 1 + 2R_n\{g_i + (g_n/R_n+g_i^2)^{1/2}\} = 1 + 2\{[4kT_o/R_{Ch} + I_{Ch}^2 S_{\delta R/R}]/4kT_o|Y_{tr}|^2\}\{g_i + [|Y_{tr}|^2(4kT_o g_i + I_G^2 S_{\delta RG/RG})/(4kT_o/R_{Ch} + I_{Ch}^2 S_{\delta R/R}) + g_i^2]^{1/2}\} \quad (3.2)$$

is obtained for

$$g_s=g_{sOPT}=(g_i^2+g_n/R_n)^{1/2}=\{g_i^2+|Y_{tr}|^2(4kT_o g_i+I_G^2 S_{\delta RG/RG})/(4kT_o/R_{Ch}+I_{Ch}^2 S_{\delta R/R})\}^{1/2} \quad (3.3)$$

Note that the present results assume a negligible direct input-output coupling (and therefore uncorrelated fluctuations in the gate and channel resistances), as can be obtained at low frequencies, and for low-frequency noise processes leading to generation of phase noise close to the carrier at high and ultra-high frequencies. It is easy to relax this neglect of correlation.

3.2 Quantum 1/f Noise Sources in GaN HFETs

The ***gate*** leakage current results in relatively large noise, and can contribute considerably to phase noise. This large contribution is caused by the piezoelectric quantum 1/f effect affecting currents present in sufficiently large samples of spontaneously polarized ferroelectric material, with a 10^6 times larger quantum 1/f noise coefficient introduced by the speed of sound replacing the speed of light in Sommerfeld's fine structure constant. It is enhanced also because gate potential fluctuations appear amplified in the channel current. Our theory yields a power spectrum of gate resistance fluctuations [5], [6],

$$S_{\delta RG/RG} = \alpha_G/N_G f;\ \alpha_G \approx 0.2 = \text{piezo Q1/f coefficient.} \tag{3.4}$$

Here $\mathbf{N_G}= I_G\tau$, where I_G is the gate current and τ is the life time of the carriers in the AlGaN gate insulation. The graph in Fig. 4.1 on p. 48 of [6], has $g\approx3$, thus $\alpha\approx0.2$.

The noise present in the GaN/AlGaN HFET's ***channel*** was derived in [5] and is used in Eq. (3. Including also the source and drain resistances, in terms of the HFETs channel resistance R_{Ch} and of the current I_{Ch} through it, we obtain $R_S+R_{Ch}+R_D=1/G_m$;

$$S_R = S_{RS}+S_{RD}+S_{Rch} +4kTR_{ch}/I_{ch}^2 = 4kTR_{ch}/I_{ch}^2 +R_S^3e\mu_S\alpha_S/fL_S^2+R_D^3e\mu_D\alpha_D/fL_D^2 \tag{3.5}$$

3.3 Total 1/f-Type Phase Noise S_y

In terms of R, R_G, I_{Ch}, and I_G, with $R_S+R_{Ch}+R_D=R$, our goal is to find the total quantum 1/f frequency fluctuation noise S_y^H=Const/f $=S_{\delta\omega/\omega}$, including also all thermal noise sources at the end. The quantum 1/f resistance fluctuations in the channel, source, drain and gate are independent, because they are fluctuations of different scattering cross sections and process rates. According to Eq. (3.5), we obtain, by including the fluctuations of the channel resistance induced by the gate resistance fluctuations,

$$S_R=S_{Rs}+S_{RD}+S_{Rch}+4kT(g_n+g_s)/(g_i+g_s)^2I_{Ch}^2)|R_{Ch}Y_{tr}|^2 \tag{3.6}$$

$$=R_S^3e\mu_S\alpha_S/fL_S +R_D^3e\mu_D\alpha_D/fL_D +4kTR_{ch}/I_{ch}^2$$

$$+\frac{1}{\mu f I_{Ch}}\left[\frac{\alpha_H}{qZ^2D_{eff}^2V_{th}}+\frac{\alpha_H^{conv}qv_s^2R_{Ch}}{I_{ch}}\xi\Theta(\xi)\right]+$$

$$+4kT(g_n+g_s)/(g_i+g_s)^2I_{Ch}^2)|R_{Ch}Y_{tr}|^2. \tag{3.7}$$

Here α_H is given by Eqs. (1.3) and (2.2), and we included the noise conductance of the gate $g_n= g_i+I_G^2S_{\delta RG/RG}/4kT$. We are now writing the resulting phase noise. With

$$S_y^H=\frac{R_{Ch}^{-2}}{4Q^4}\{\frac{1}{\mu f I_{Ch}}[\frac{\alpha_H}{qZ^2D_{eff}^2V_{th}}+\frac{\alpha_H^{conv}qv_s^2R_{Ch}}{I_{Ch}}\xi\Theta(\xi)]+\frac{\alpha_s}{fN_s}R_s^2+\frac{\alpha_D}{fN_D}R_D^2$$

$$+\frac{4kT_o(g_s+g_n)}{(g_s+g_i)^2I_{Ch}^2}|R_{Ch}Y_{tr}|^2+4kTR_{Ch}/I_{Ch}^2\};\ldots.\xi\equiv 1-\frac{V_G^*}{R_{Ch}I_{Ch}}+\frac{V_{th}}{R_{Ch}I_{Ch}};$$

$$\Theta(\xi)\equiv(1+\text{sign}\xi)/2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.8)$$

$y \equiv \delta\omega/\omega$, the above equations (3.8) provide the quantum 1/f frequency fluctuations component, which is dominant close to the carrier, arising from the GaN/AlGaN HFETs. They also yield the frequency fluctuation from thermal noise in the gate and the channel. To this result we have to add the well-known contribution of the sidebands arising from device nonlinearity (NSB-part).

This NSB part can be reduced by operating the HFET at saturation, and by restricting the dynamical range. Furthermore, partial compensation measures are possible for this classical part.

The phase noise at a frequency $\Delta\nu=f$ from the carrier frequency is given most often not in terms of frequency fluctuations, but in dBc, i.e.,

$$\mathcal{L}(f)= \mathcal{L}(\Delta\nu)=10\log[(1/2)(\omega/2\pi f)^2 S_y(f)] \tag{3.9}$$

Nanotechnology raises new questions of electronic noise, since fluctuations are more important in smaller devices. Based on the quantum 1/f noise theory, we find that in a certain transition range of device sizes this general law is suspended, but reappears for 1/f noise in the nanometer domain, where the transition from coherent to conventional quantum 1/f effect is complete [5] causing Moore's Law to fail. The resulting quantum 1/f formulas are used to describe 1/f noise in materials, devices and systems. They are used here to calculate the phase noise of GaN/AlGaN HFET-based oscillators. RTDs, BAW and SAW quartz resonators, MEMS resonators, and spin valves, or in general in spintronic devices, are considered elsewhere [5]. They are also used to calculate phase noise in these devices and in oscillators based on them [5], from first principles along with some classical noise sources. Device optimization is thus facilitated for ultra-small devices, and in general, for any kind of Hi-tech material, device, or system. This is important for radar, radio-links, wireless communication in general, both civil and military.

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