# 1/f Noise in the Dark Current of GaN QWIPs

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## ABSTRACT

The dark current of Quantum Well Inter-subband Photodetectors is affected by 1/f noise that limits the detectivity. This paper applies for the first time conventional quantum 1/f noise expressions to calculate the expected level of 1/f noise in QWIPs and applies the resulting engineering formulas to the case of GaInAs/InP and GaN/AlGaN QWIPs. Both the collisionless and collision-dominated cases are considered. The elementary process causing the dark current is the transfer of an electron from one well to the neighboring well. This happens under the influence of the applied electric field, and has in general both thermally activated and tunneling components. The larger the applied electric field, the larger is the squared velocity change of the carriers, and the larger is the obtained conventional quantum 1/f effect. The detectivity of the devices is calculated on this basis. Quantum well intersubband photodetectors (QWIPs) can be extended in principle from infrared into the THz region.

Keywords: Quantum 1/f noise, QWIPs, Photodetectors, Quantum wells, Conventional quantum 1/f effect, Dark current, 1/f noise, detectivity. Quantum Well Intersubband Photodetectors.

## 1. INTRODUCTION

Most of the observed ubiquitous 1/f noise[1] is fundamental, in the form of quantum 1/f noise[2-23], and the dark current in QWIPs is no exception. Quantum 1/f noise is the manifestation of the quantum 1/f effects, a new aspect of quantum mechanics. Quantum 1/f effects (Q1/fE) characterize the form in which electrical charges, and matter (gravidynamical Q1/fE) in general, exist in space and time. They arise from the nonlinearity caused by interaction of electrical charges with their field. The Q1/fE are therefore as fundamental as the notion of scattering, or as the notion of interaction (or "existence"). They include the conventional[2-20] and coherent[20, 9-12,14-16, 18, 20] Q1/fE. The first was derived also from the quantum information theory[17], and the second from the asymptotic form of the only physical electrodynamic propagator[15, 14, 12], which has a branch-point singularity.

Although quantum 1/f noise is epistemologically a special case of fundamental 1/f noise, being caused by the simultaneous presence of nonlinearity and homogeneity in quantum electrodynamics at low frequencies, as defined by the universal sufficient criterion[23], it is the most fundamental from the ontological point of view. That is the point of view that constructs the universe starting from quarks and leptons. In particular, the Q1/fE is more fundamental than thermal noise and modulates[4, 19] the level of Nyquist (or Johnson) noise in current or voltage (but not in available power) to yield the physical thermal noise.

Below, the Q1/fE are defined and derived in physical terms with references to more rigorous derivations. The conventional Q1/fE is then applied to various scattering mechanisms in homogeneous semiconductors, to GaInAs/InP and GaN QWIPs.

## 2. CONVENTIONAL AND COHERENT QUANTUM 1/F EFFECTS

### 2.1. Conventional quantum 1/f effect

This effect[2-23], is present in any cross section or process rate involving charged particles or current carriers. The physical origin of quantum 1/f noise is easy to understand. Consider for example Coulomb scattering of current carriers, e.g., electrons on a center of force. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by DeBroglie waves of a frequency corresponding to their energy. However, some of the electrons have lost energy in the scattering process, due to the emission of bremsstrahlung. Therefore, part of the outgoing DeBroglie waves is shifted to slightly lower frequencies. When we calculate the

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probability density in the scattered beam, we obtain also cross terms, linear both in the part scattered with and without bremsstrahlung. These cross terms oscillate with the same frequency as the frequency of the emitted bremsstrahlung photons. The emission of photons at all frequencies results therefore in probability density fluctuations at all frequencies. The corresponding current density fluctuations are obtained by multiplying the probability density fluctuations by the velocity of the scattered current carriers. Finally, these current fluctuations present in the scattered beam will be noticed at the detector as low frequency current fluctuations, and will be interpreted as fundamental cross section fluctuations in the scattering cross section of the scatterer. While incoming carriers may have been Poisson distributed, the scattered beam will exhibit super-Poissonian statistics, or bunching, due to this new effect which we may call quantum 1/f effect. The quantum 1/f effect is thus a many-body or collective effect, at least a two-particle effect, best described through the two-particle wave function and two-particle correlation function.
Let us estimate the magnitude of the quantum 1/f effect semiclassically. We start with the classical (Larmor) formula $2q^2\mathbf{a}^2/3c^3$, for the power radiated by a particle of charge q and acceleration **a**. The acceleration can be approximated by a delta function $\mathbf{a}(t) = \mathbf{\Delta v}\delta(t)$ whose Fourier transform $\mathbf{\Delta v}$ is constant and is the change in the velocity vector of the particle during the almost instantaneous scattering process. The one-sided spectral density of the emitted bremsstrahlung power $4q^2(\mathbf{\Delta v})^2/3c^3$ is therefore also constant. The number $4q^2(\mathbf{\Delta v})^2/3hfc^3$ of emitted photons per unit frequency interval is obtained by dividing with the energy hf of one photon. The probability amplitude of photon emission $[4q(\mathbf{\Delta v})^2/3hfc^3]^{1/2}e^{i\gamma}$ is given by the square root of this photon number spectrum, including also a phase factor $e^{i\gamma}$. Let $\psi$ be a representative Schrödinger catalogue wave function of the scattered outgoing charged particles, which is a single-particle function, normalized to the actual scattered particle concentration. The beat term in the probability density $\rho=|\psi|^2$ is linear both in this bremsstrahlung amplitude and in the non-bremsstrahlung amplitude. Its spectral density will therefore be given by the product of the squared probability amplitude of photon emission (proportional to 1/f) with the squared non-bremsstrahlung amplitude which is independent of f. The resulting spectral density of fractional probability density fluctuations is obtained by dividing with $|\psi|^4$ and is therefore

$$|\psi|^{-4}S_{|\psi|^2}(f) = 8q^2(\mathbf{\Delta v})^2/3hfNc^3 = 2\alpha A/fN = j^{-2}S_j(f), \qquad (2.1)$$

where $\alpha = e^2/\hbar c = 1/137$ is the fine structure constant and $\alpha A = 4q^2(\mathbf{\Delta v})^2/3hc^3$ is known as the infrared exponent in quantum field theory, and is known as the quantum 1/f noise coefficient, in electrophysics. The experimentally measured quantity also depends on many superposition or averaging processes and is often called, for historical reasons, Hooge constant.
The spectral density of current density fluctuations is obtained by multiplying the probability density fluctuation spectrum with the squared velocity of the outgoing particles. When we calculate the spectral density of fractional fluctuations in the scattered current j, the outgoing velocity simplifies, and therefore Eq. (2.1) also gives the spectrum of current fluctuations $S_j(f)$, as indicated above. The quantum 1/f noise contribution of each carrier is independent, and therefore the quantum 1/f noise from N carriers is N times larger. However, the current j will also be N times larger, and therefore in Eq. (2.1) a factor N was included in the denominator, for the case in which the cross section fluctuation is observed on N carriers simultaneously.
The fundamental fluctuations of cross sections and process rates are reflected in various kinetic coefficients in condensed matter, such as the mobility $\mu$ and the diffusion constant D, the surface and bulk recombination speeds s, and recombination times $\tau$, the rate of tunneling $j_t$ and the thermal diffusivity in semiconductors. Therefore, the spectral density of fractional fluctuations in all these coefficients is given also by Eq. (2.1).
When we apply Eq. (2.1) to a certain device, we first need to find out which are the cross sections $\sigma$ or process rates that limit the current I through the device, or that determine any other device parameter P. Then we have to determine both the velocity change $\Delta\mathbf{v}$ of the scattered carriers and the number N of carriers simultaneously used to test each of these cross sections or rates. Further, Eq. (2.1) provides the spectral density of quantum 1/f cross section or rate fluctuations. These spectral densities are multiplied by the squared partial derivative $(\partial I/\partial\sigma)^2$ of the current, or of the device parameter P of interest, to obtain the spectral density of fractional device noise contributions from the cross sections and rates considered. After doing this with all cross sections and process rates, we add the results and bring (factor out) the fine structure constant $\alpha$ as a common factor in front. This yields excellent agreement with the experiment[1, 10, 11, 13], in a large variety of samples, devices and physical systems[24-27].
Eq. (2.1) was also derived in second quantization[5, 9, 12, 14, 15, 18, 22, 23], using the commutation rules for boson field operators. For fermions one repeats the calculation replacing in the derivation the commutators of field operators by anticommutators, which yields

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/f(N-1). \tag{2.2}$$

This causes no difficulties, since $N\geq2$ for particle correlations to be defined, and is practically the same as Eq. (2.4), since usually $N>>1$. Eqs. (2.1) and (2.2) suggest a new notion of physical cross sections and process rates which contain 1/f noise, and express a fundamental law of physics, important in most high-technology applications.

We conclude that the conventional quantum 1/f effect can be explained in terms of interference beats. The interference is between the part of the outgoing DeBroglie waves scattered without bremsstrahlung energy losses above the detection limit (given in turn by the reciprocal duration T of the 1/f noise measurement) on one hand, and the various parts scattered with bremsstrahlung energy losses. There is more to it than that: exchange between identical particles is also important. This, of course, is just one way to describe the reaction of the emitted bremsstrahlung back on the scattered current. This reaction, itself an expression of the nonlinearity introduced by the coupling of the charged-particle field to the electromagnetic field, thus reveals itself as the cause of the quantum 1/f effect, and implies that the effect can not be obtained with an independent boson model. The effect, just like the classical turbulence-generated 1/f noise[9, 23], is a result of the scale-invariant nonlinearity of the equations of motion describing the coupled system of matter and field. Ultimately, therefore, this nonlinearity is the source of the 1/f spectrum in both the classical and quantum form of the theory. We can say that the quantum 1/f effect is an infrared divergence phenomenon, this divergence being the result of the same nonlinearity. The quantum 1/f effect is, in fact, the first time-dependent infrared radiative correction. Finally, it is also deterministic in the sense of a well determined wave function, once the initial phases $\gamma$ of all field oscillators are given. In quantum mechanical correspondence with its classical turbulence analog[9, 23], the new effect is therefore a quantum manifestation of classical chaos which we can take as the definition of a certain type of quantum chaos.

### 2.2. Elementary application to homogeneous semiconductor samples.

In a homogeneous sample of length L, cross section A, volume V=AL, carrier mobility $\mu$, carrier concentration n and total number of carriers N=nAL, the conductance $C=n\mu eA/L$ and the resistance R=1/C will exhibit quantum 1/f fluctuations with a spectral density $S_{\delta C/C}$ of fractional fluctuations $\delta C/C = -\delta R/R$ given by

$$S_{\delta C/C}(f) = S_{\delta R/R}(f) = S_{\delta\mu/\mu}(f) = \alpha_o/fN. \tag{2.3}$$

Size dependence. To calculate $\alpha_o$ we first evaluate the parameter $s = nA\cdot5.5\cdot10^{-13}$ cm introduced in Sec. 2.4 below. If $s\geq1$, coherent quantum 1/f noise (see Sec. 2.3 below) is observed with $\alpha_o= 2\alpha/\pi$.

If s<1, Eq. (2.6) requires knowledge of $(\alpha_o)_{conv}$. The latter is calculated from Eq. (2.1) or (2.2) for each type of scattering which limits the mobility $\mu = e\tau/m^*$ of the carriers. Here $\tau$ is the mean collision time or scattering time of the carriers and $m^*$ is their effective mass. In terms of the mean frequency of collisions $\nu = 1/\tau = \sigma v n_i$, one obtains $\mu = e/\nu m^* = e/\sigma v n_i m^*$. Here v is the mean speed of the carriers between collisions, $\sigma$ a scattering cross section and $n_i$ the concentration of scatterers.

**Conventional Q1/fE in the mobility**. In general, Matthiessen' rule allows us to write, in terms of mobility,

$$1/\mu = \sum_j 1/\mu_j \quad , \tag{2.4}$$

where $\square_j$ is the mobility which would be obtained if only the j-th scattering mechanism would be present and would limit the mobility. Applying a quantum 1/f fluctuation to Eq. (2.7), squaring and averaging quantum mechanically and statistically, we obtain as a reasonable first approximation

$$\delta\mu/\mu^2 = \sum_j \delta\mu_j/\mu_j^2 \quad ; S_{\delta\mu/\mu}(f) = \sum_j (\mu/\mu_j)^2 S_{\delta\mu/\mu_j}(f). \tag{2.5}$$

Eq. (2.2) yields the strongest conventional quantum 1/f noise for umklapp scattering, followed by the f and g forms of inter-valley scattering or inter-valley with umklapp (in indirect band gap semiconductors like Si and Ge only), followed by normal phonon scattering, by neutral impurity scattering and by ionized impurity scattering (that has lowest 1/f noise). The corresponding terms in Eq. (2.5) reflect this hierarchy only partially, because of the factors $(\mu/\mu_j)^2$ which gauge the importance of each of the scattering processes in limiting the resultant mobility. To gain physical insight, the conventional quantum 1/f effect present in the various scattering processes is just estimated below and is actually calculated in Sec. 4.4 by taking into account the corrections introduced by the momentum distribution of the carriers and by the phonon distribution function at the temperature T.

**Impurity scattering.** For instance, in the case of impurity scattering, $n_i$ is. One obtains $S_{\delta\mu/\mu}(f) = S_{\delta\sigma/\sigma}(f)$. The physical scattering cross section $\sigma$, in turn, exhibits the Q1/fE with the spectral density given by Eqs. (2.1) and (2.2)

$$S_{\delta\sigma/\sigma}(f)_i = (4\alpha/3\pi fN)<[\mathbf{\Delta v}/c]^2> \approx (3\cdot 10^{-3}/fN)(\hbar\, \Delta\mathbf{k}/m^* c)^2. \qquad (2.6)$$

The average quadratic velocity change of the electrons in a scattering process is smaller in impurity scattering than in lattice scattering which includes normal phonon scattering, inter-valley scattering in indirect band gap semiconductors with several "valleys", and umklapp scattering, as well as optical phonon scattering. The Coulomb/Rutherford or Conwell/Weiskopf scattering cross section is proportional to $1/|\Delta\mathbf{k}|^4$, which favors small angle scattering. Nevertheless, there are a few larger angle scattering events which are most effective in limiting the mobility and which therefore are decisive in the exact evaluation of the Q1/fE coefficient as a slow function of $n_i$, the concentration of impurities, given in the theory section below. This corresponds approximately to assuming randomizing collisions,

$$<(\mathbf{\Delta v})^2>_i = 2<v^2> = 6k_B T/m^*, \qquad (2.7)$$

although impurity scattering is not randomizing. With $m_o$ representing the free electron mass, we obtain this way

$$S_{\delta\mu/\mu}(f)_i = S_{\delta\sigma/\sigma}(f)_i = (4\alpha/3\pi fN)<[6k_B T/m^* c^2]> \approx (10^{-9}/fN)(Tm_o/4m^* 100K). \qquad (2.8)$$

The quantum 1/f noise power present in impurity scattering is therefore proportional to T.

**Normal acoustic phonon scattering.** For normal phonon scattering, the product $\sigma v n_i$ has to be replaced by the lattice scattering rate $\Gamma$, given by an effective number of phonons times the squared phonon scattering matrix element, $|M|^2$. The latter exhibits quantum 1/f fluctuations, because the carriers emit bremsstrahlung photons in the scattering process. Therefore, if the mobility $\mu$ of electrons (of random velocity $v=\hbar k/m^*$) is limited by phonon scattering, we get

$$S_{\delta\mu/\mu}(f)_{ap} = S_{\delta\Gamma/\Gamma}(f)_{ap} = (4\alpha/3\pi fN)<(\hbar\, \Delta\mathbf{k}/m^* c)^2> = (3\cdot 10^{-3}/fN)<(\hbar\, \Delta\mathbf{q}/m^* c)^2>. \qquad (2.9)$$

Here $\Delta\mathbf{q}$ is the acoustic phonon momentum transfer in the scattering process and the brackets indicate the average value. Using the linear approximation of the acoustic phonon dispersion relation $E_q=v_s q\hbar$, with $v_s$ denoting the speed of sound, we obtain for a thermal phonon with $E_q=k_B T/2$

$$<(\hbar\, \Delta\mathbf{q}/m^* c)^2> = (k_B T/2v_s m^* c)^2 = 1.25\cdot 10^{-5}\,(m_o/m^*)^2 \qquad (2.10)$$

We finally obtain

$$S_{\delta\mu/\mu}(f)_{ap} = (3.75\cdot 10^{-8}/fN)(m_o/m^*)^2. \qquad (2.11)$$

The mean square momentum change and the 1/f noise are much larger (e.g., 50 times) for acoustic phonon scattering, because impurity scattering is mainly small-angle scattering. A more rigorous treatment for the many types of scattering present in semiconductors, taking into account the corrections introduced by the momentum distribution of the carriers and the phonon distribution function at the temperature T is given in the theory section below in the second half of this article for the case of silicon.

**Umklapp scattering.** In this case the momentum change is close to the smallest reciprocal lattice vector approximated by $\hbar G=2\pi\hbar/a$, where a is the lattice constant. Therefore, the first form of Eq. (2.10) yields

$$(\alpha_{oU})_{conv} = (4\alpha/3\pi)[2\pi\hbar/am^* c]^2 = (6\cdot 10^{-8}/fN)(m_o/m^*)^2 \qquad (2.12)$$

for Umklapp scattering.

**Inter-valley scattering.** In indirect band gap semiconductors the location of the energy minima of the conduction band in k-space is different from the location of the valence band energy maxima. In thermal equilibrium electrons and holes are present close to the minima of the conduction and valence bands. Scattering processes carrying electrons from one minimum (or valley) to the other are known as inter-valley scattering. This is large-angle scattering, compared with normal (intra-valley) scattering, and it is therefore affected by larger conventional quantum 1/f noise, almost as large as Umklapp scattering. Indeed, e.g., in Si the 8 minima are located at 0.85G from the origin, where G is the smallest reciprocal lattice vector. For g-processes which scatter an electron to the valley symmetrically located on the other side of the origin, Eq. (2.12) remains valid with a correction factor of $(0.85)^2$. On the other hand for f-processes in Si, scattering electrons between to neighboring valleys, the factor is 2 times smaller. There is also the possibility of inter-valley scattering with umklapp which requires a correction factor of $(1-0.85)^2$. Eq. (2.12) thus yields for inter-valley scattering with Umklapp

$$(\alpha_{oIU})_{conv} = 0.0225\,(4\alpha/3\pi)[2\pi\hbar/am^*c]^2 = (1.35\cdot 10^{-9}/fN)(m_o/m^*)^2. \tag{2.13}$$

While this appears to indicate a lower contribution from these inter-valley umklapp processes, the corresponding $(\mu/\mu_j))^2$ factor in Eq. (2.8) insures a larger contribution to the resulting spectral density of quantum 1/f noise $S_{\delta\mu/\mu}(f)$. Physically, this is caused by the scarcity of high-energy phonons able to bridge the momentum gap of 0.85 G.

### 2.3 Physical derivation of the coherent quantum 1/f effect

This effect arises in a beam of electrons (or other charged particles propagating freely in vacuum) from the definition of the physical electron as a bare particle plus a coherent state of the electromagnetic field[20]. It is caused by the energy spread characterizing any coherent state of the electromagnetic field oscillators, an energy spread which spells non-stationarity, i.e., fluctuations. To find the spectral density of these inescapable fluctuations which are known to characterize any quantum state which is not an energy eigenstate, we use an elementary physical derivation based on Schrödinger's definition of coherent states. This supplements a rigorous derivation[12, 14, 15], which was given from a well-known physical, quantum-electrodynamical, propagator.

The coherent quantum 1/f effect will be derived[20] in three steps: first we consider a hypothetical world with just one single mode of the electromagnetic field coupled to a beam of charged particles; considering the mode to be in a coherent state, we calculate the autocorrelation function of the quantum fluctuations in the particle-density (or concentration) which arise from the nonstationarity of the coherent state. Then we calculate the amplitude with which this one mode is represented in the field of an electron, according to electrodynamics. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the electromagnetic field be characterized by the wave vector **q**, the angular frequency $\omega = cq$ and the polarization $\lambda$. Denoting the variables **q** and $\lambda$ simply by q in the labels of the states, we write the coherent state of amplitude $|z_q|$ and phase $\arg z_q$ in the form

$$|z_q\rangle = \exp[-(1/2)|z_q|^2]\exp[z_q a_q^+]\,|0\rangle = \exp[-(1/2)|z_q|^2][\sum_{n=0}^{\infty} z_q^{\,n}/\sqrt{n!}]\,|n\rangle. \tag{2.14}$$

Here $a_q^+$ is the creation operator which adds one energy quantum to the energy of the mode q. Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n\rangle = (2^n n!\sqrt{\pi})^{-1/2}\exp[-x^2/2]\,H_n(x)\,e^{in\omega t}. \tag{2.15}$$

This yields for the coherent state $|z_q\rangle$ the representation

$$\Psi_q(x) = \exp[-(1/2)|z_q|^2]\exp[-x^2/2]\sum_0^{\infty}\frac{[z_q e^{i\omega t}]^n H_n(x)}{[n!(2^n\sqrt{\pi})]^{1/2}} = \exp[-|z_q|^2/2]\exp[-x^2/2]\exp[-z_q^2 e^{-2i\omega t} + 2xz_q e^{i\omega t}]. \tag{2.16}$$

In the last form the generating function of the Hermite polynomials was used. The summation extends to infinity at the upper limit. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time t or the phase of $z_q$, is, for $|z_q|<<1$,

$$P_q(\tau,x) = <|\Psi_q|_t^2 \mid \Psi_q|_{t+\tau}^2> = \{1 + 8x^2|z_q|^2[1 + \cos\omega\tau] - 2|z_q|^2\}\exp[-x^2/2]. \tag{2.17}$$

Integrating over x from $-\infty$ to $\infty$, we find the autocorrelation function

$$A^1(\tau) = (2)^{-1/2}\{1 + 2|z_q|^2\cos\omega\tau\}. \tag{2.18}$$

This result shows that the probability distribution contains a constant background with small superposed oscillations of frequency $\omega$. Physically, the small oscillations in the total probability describe self-organization or bunching of the particles in the beam. They are thus more likely to be found in a measurement at a certain time and place than at other times and places relative to each other along the beam. Note that for $z_q = 0$ the coherent state becomes the ground state of the oscillator which is also an energy eigenstate, and therefore stationary and free of oscillations.
We now determine the amplitude $z_q$ with which the field mode q is represented in the physical electron. One way to do this is to let a bare particle dress itself through its interaction with the electromagnetic field, i.e. by performing first order perturbation theory with the interaction Hamiltonian

$$H' = A_\mu j^\mu = -(e/c)\mathbf{v}\cdot\mathbf{A} + e\phi, \tag{2.19}$$

where **A** is the vector potential and $\phi$ the scalar electric potential. Another way is to Fourier expand the electric potential e/r of a charged particle in a box of volume V. In both ways we obtain the well-known result

$$|z_q|^2 = \pi(e/q)^2(\hbar\, cqV)^{-1}. \tag{2.20}$$

Considering now all modes of the electromagnetic field, we obtain from the single - mode result of Eq. (2.18)

$$A(\tau)= C\,\Pi_q\{1 + 2|z_q|^2\cos\omega_q\tau\} = C\{1 + \Sigma_q 2|z_q|^2\cos\omega_q\tau\}= C\{1 + 4(V/2^3\pi^3)\int d^3q\,|z_q|^2\cos\omega_q\tau\} \tag{2.21}$$

Here we have again used the smallness of $z_q$ and we have introduced a constant C. Using Eq. (2.23) we obtain

$$A(\tau) = C\{1 + 4\pi(V/2^3\pi^3)(4\pi/V)(e^2/\hbar\, c)\int(dq/q)\cos\omega_q\tau\}= C\{1 + 2(\alpha/\pi)\int\cos(\omega\tau)d\omega/\omega\}. \tag{2.22}$$

Here $\alpha = e^2/\hbar c$ is Sommerfeld's fine structure constant 1/137. The first term in curly brackets is unity and represents the constant background, or the d.c. part of the current carried by the beam of particles through vacuum. The autocorrelation function for the relative (fractional) density fluctuations, or for the current density fluctuations in the beam of charged particles is obtained therefore by dividing the second term in curly brackets by the first term. The constant C drops out when the fractional fluctuations are considered. According to the Wiener-Khintchine theorem, the coefficient of $\cos\omega\tau$ is the spectral density of the fluctuations, $S_{|\psi|^2}$ for the particle concentration, or $S_j$ for the current density $j = e(k/m)|\psi|^2$

$$S_{|\psi|^2}<|\psi|^{-2}> = S_j<j>^{-2} = 2(\alpha/\pi f N) = 4.6\cdot10^{-3}\, f^{-1}N^{-1}. \tag{2.23}$$

Here we have included the total number N of charged particles which are observed simultaneously in the denominator, because the noise contributions from each particle are independent. This result is related to the conventional Quantum 1/f Effect introduced above. Eq. (2.23) was also derived directly[12, 14, 15], from the well-known physical QED propagator of Kulish, Faddeev and Zwanziger.

### 2.4 Relation between coherent and conventional quantum 1/f effect.

We turn now to the connection[28] with the conventional Quantum 1/f Effect. The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy $E_m$, per unit length, of the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int(B^2/8\pi)d^3x = [nevS/c]^2[1/4 + \ln(R/r)] \qquad (2.24)$$

is much smaller than the total kinetic energy $E_k$ of the drift motion of the individual carriers

$$E_k = \Sigma mv^2/2 = nSmv^2/2 = E_m/s. \qquad (2.25)$$

Here we have introduced the magnetic field B, the carrier concentration n, the cross sectional area S and radius r of the cylindrical sample (e.g., a current carrying wire), the radius R of the electric circuit, and the "coherence ratio"

$$s = E_m/E_k = 2ne^2S/mc^2[1/4 + \ln(R/r)] \approx 2e^2N'/mc^2 = 2N'r_o = 5.68{\cdot}10^{-13}\text{cm}{\cdot}N', \qquad (2.26)$$

where N' = nS is the number of carriers per unit length of the sample, $r_o = e^2/mc^2 = 2.84{\cdot}10^{-13}$ cm is the classical radius of the electron, and the expression in rectangular brackets has been approximated by unity in the last form. We expect the total observed spectral density of the mobility fluctuations to be approximated by a relation of the form

$$(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN] \qquad (2.27)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side. In this equation $\alpha A=2\alpha(\Delta\mathbf{v}/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum 1/f effect. This equation is limited to quantum 1/f mobility (or diffusion) fluctuations, and does not include the quantum 1/f noise in the surface and bulk recombination cross sections, in the surface and bulk trapping centers, in tunneling and injection processes in emission or in transitions between two solids.
two solids.

Note that the coherence ratio s introduced here equals the unity for the critical value N' =N" = $2{\cdot}10^{12}$/cm., e.g. for a cross section S = $2{\cdot}10^{-4}$ cm$^2$ of the sample when n = $10^{16}$. For small samples with N'<<N" only the first term survives, while for N'>N" the second term in Eq. (2.27) is dominant.

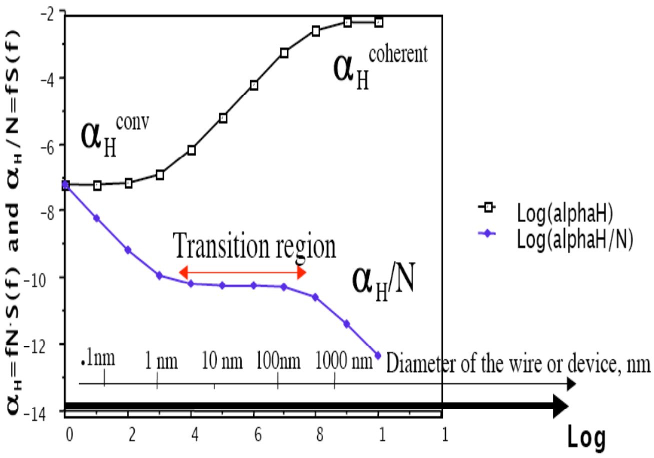


Fig. 1: The Q1/f parameter $\alpha_H$ and spectrum of fractional mobility fluctuations $fS_{\delta\mu/\mu}\equiv\alpha_H/N$ shown as a function of the number N of carriers in the sample.

The transition region, in which we observe the proximity effect, is marked by a red arrow in Fig. 1. The wire diameter scale gets shifted to the right (or left) proportionally when the concentration of carriers increases (or decreases) from the assumed value of $10^{17}$ cm$^{-3}$. We took the cross section to be (140nm$^2$), the length 5 mm, and the concentration n = $10^{17}$cm$^{-3}$as an example. The flat portion of the lower curve explains why 1/f noise was less of a problem during the past miniaturization stage. It is now again a problem, as the increase below logN=3 shows.

***The case of FETs/HFETs***. Finally, an important special case should be mentioned. Eq. (2.24) was derived for the case of a cylindrical sample with circular cross section of radius r, shaped like a straight wire. Modern FETs and HFETs try to increase the denominator N in Eq. (2.27) without markedly increasing the coherence parameter s, by choosing an extreme case of a rectangular cross section for the portion between source and drain, and even more extreme for the channel, which is the region under the gate. Indeed, the circular cross section is replaced by a rectangle with a very large width w of several hundreds of microns and less than a micron thick, the channel being even thinner. The channel is reduced to a 2-dimensional quantum sheet, which is a quantum well of size (thickness) t in the 3rd dimension, towards the gate. Most important, the "length" $L_{DS}$ of the source-to-drain distance is only of the order of 1micron, while the gate length is just about a quarter micron or less. In this case the kinetic energy $E_k$ per unit length in the direction of $L_{DS}$ is still given by Eq. (2.25), but the magnetic energy $E_m$ per unit length in the direction of $L_{DS}$ is roughly proportional with the first power of w only, instead of $w^2$, and can be approximated by

$$E_m = \pi[\ln(w/2L_{DS})]L_{DS}[nevS/c]^2/w. \tag{2.28}$$

This indicates decoherence along the device width w. For $w>>L_{DS}>t$, this yields a coherence ratio

$$s \approx \pi n r_o t L_{DS} \ln(w/2L_{DS}) \qquad \text{for } w>>L_{DS}>t, \tag{2.29}$$

which shows that only an effective width $w=w_{eff}$ about equal to $L_{DS}$ should be used in the calculation of the coherence parameter s in this special case; larger widths are subject to de-coherence, i.e., their magnetic energy no longer increases proportional to $w^2$, or $N'^2$. This favors lower, mainly conventional, quantum 1/f noise in these devices, in spite of the large values of w, in agreement with the practical results, which anticipated the quantum 1/f result simply by trial and error, unaware of the subtleties of the transition from coherent to conventional Q1/f noise in this case, and in general. It also shows that fragmentation of the extremely large widths w does not reduce 1/f noise in this special case.

*Note:* To derive the rough approximation given in Eq. (2.28), one writes the expression of the coherent ($\propto n^2$) contribution of the currents in the short, but very wide channel to the magnetic energy of the circuit in the form

$$E_m=\int[\mathbf{j}(\mathbf{x})\cdot\mathbf{j}(\mathbf{x'})/2c^2|\mathbf{x}-\mathbf{x'}|]d^3x d^3x'=e^2v_d^2\int[n(\mathbf{x})\cdot n(\mathbf{x'})/2c^2|\mathbf{x}-\mathbf{x'}|]d^3x d^3x'. \tag{2.30}$$

The integral coincides with the electrostatic energy of an equivalent constant charge density numerically equal to $e/c^2$ in CGS, or $e\mu_o/4\pi\varepsilon_0$ in SI, uniformly distributed over the volume $wtL_{DS}$ of the channel. This energy, in turn, is calculated as the corresponding volume integral over the squared equivalent electric field, using $L_{DS}$ as a lower cut-off. The equivalent electric field is $netw/cr(r^2+w^2/4)^{1/2}$ at a distance r in the median plane of a segment of length w oriented along the width w of the channel. A more elaborate calculation does not simply approximate the field with its value in this median plane, but yields similar results in the end.

## 3. APPLICATION TO QWIPS

The Quantum Well Intersubband Photodetector is a semiconductor photon detector that responds to absorption within its bandgaps. The structure is designed to allow an excited carrier within the well to escape, joining others to form what is called the dark current. Because the dark current is a factor in optimizing device performance, we will investigate the noise associated with this current. The noise is described by the conventional quantum 1/f effect[2-23] which describes the fundamental fluctuations in the physical cross sections and process rates that generate the dark current. The calculation of the 1/f noise is then compared to experimental data from Jutao Jiang[29]. A similar calculation was performed by us earlier for n+p junction infrared detectors and MIS infrared detectors[30].

### 3.1Theory

Applying an electric field to the quantum well accelerates the electrons, and the resulting drift velocity gives rise to the dark current. Conservation of energy gives an expression for this velocity

$$\frac{m}{2}(\Delta\upsilon)^2 = (L_b + L_w)eE + |W_0|, \tag{3.1}$$

where m is the electron mass, $L_b$ is the barrier thickness, $L_w$ is the width of the well, E is the magnitude of the applied field and $W_0$ is the binding energy of the well. Here we assume that the carriers move under uniform acceleration between the wells and that the effects of collisions between the carriers are small enough to be ignored. The conventional quantum 1/f effect that occurs in these devices is the result of fluctuations in the capture cross section that determines the mobility and the dark current. According to Eq. (2.1), the spectral density of these current fluctuations is given by

$$S_{\frac{\delta\rho}{\rho}} = S_{\frac{\delta j_{dark}}{j_{dark}}} = \frac{4\alpha}{3\pi fN}\left(\frac{\Delta\upsilon}{c}\right)^2 = j^{-2}S_{\delta j_{dark}}(f), \tag{3.2}$$

where N is the number of carriers defining the current. Substituting the velocity change from Eq. (3.1), we obtain

$$S_{\delta j_{dark}} = \frac{4\alpha}{3\pi fN}\left(\frac{2}{mc^2}\right)\left[(L_b + L_w)eE + W_0\right] j_{dark}^2 . \tag{3.3}$$

Next, recalling the relation between the dark particle-current $j_{dark}$ and the number of carriers,

$$j_{dark} = \left\langle \frac{dN}{dx} \right\rangle \left\langle \upsilon_{carriers} \right\rangle . \tag{3.4}$$

Classically, an electron accelerated between two points has a final velocity of

$$\upsilon_{end} = \sqrt{2a(\Delta l)} \tag{3.5}$$

and we write the average velocity of the carriers as

$$< \upsilon_{carriers} > = \tfrac{1}{2}\sqrt{2a(\Delta l)} = \tfrac{1}{2}\sqrt{2a(L_b + L_w)} . \tag{3.6}$$

To simplify, we let $L = L_b + L_w$ represent the lattice constant of the quantum well and use $a = eE/m$ for the acceleration. The dark current is then

$$j_{dark} = \frac{N}{L}\frac{1}{2}\sqrt{\frac{2eEL}{m}} \tag{3.7}$$

Finally, with this relation between the number of carriers and the dark current, the spectral density can be simplified

$$S_{\delta j_{dark}} = \frac{8\alpha}{3\pi fmc^2}\left(\frac{\sqrt{eE}}{\sqrt{2Lm}}\right)\left[LeE + W_0\right] j_{dark} \tag{3.8}$$

To quantify the dark current fluctuations, we now apply the data and parameters of the GaInAs/InP QWIP fabricated and tested by Jutao Jiang [29].

Table 1. Values of the GaInAs/InP QWIP Parameters

| PARAMETER: | Value used | |
|---|---|---|
| Dark Current $j_{dark}$ at 77K | 2 | μA |
| MQW lattice period L | 570 | Å |
| Total voltage applied 20EL | 2 | V |
| Voltage applied on 1 well = EL | 0.1 | V |
| Binding energy in the well $W_0$ | 0.1 | eV |

Next, we apply the specific parameters of the QWIP device to our derived spectral density equation (3.8). The binding energy, $W_0$, for the well is 0.251eV. The combined length of the barrier and the well in the direction of the current j that is perpendicular to the well, is 571 angstroms. At low temperatures (around 70K), the dark current is around 2μA. Plugging these values shown in Table 1 into Eq. (3.8) and dividing Eq. (3.8) by the square of the current $j_{dark}$, we obtain the spectral density of its fractional fluctuations in the form

$$S_{\delta j/j}(f) = (j_{dark})^{-2}S_{\delta j} = (6.2\cdot 10^{-3}\cdot 0.2\text{eV}/f\,5.5\cdot 10^{5}\text{eV})(0.1\cdot 1.6\cdot 10^{-19}\text{J}/2\cdot 5.7^{2}\cdot 10^{-16}\text{m}^{2}\cdot 0.9\cdot 10^{-30}\text{kg})^{1/2}(1.6\cdot 10^{-19}\text{C}/2\cdot 10^{-6}\text{A}). \tag{3.9}$$

Therefore, with the effective mass of the electrons in the InP barrier $m=0.077m_o$, we get a 48 times larger result

$$S_{\delta j/j}(f) = (2.25\cdot 10^{-9}/f)(1.66\cdot 10^{12})(8\cdot 10^{-14})48 = 1.44\cdot 10^{-8}/f. \tag{3.10}$$

Amplifying Eq. (3.10) with the square of $j_{dark} = 1.25\cdot 10^{13}$/s we obtain the spectral density of dark current fluctuations

$$S_{\delta j}(f) = (2.25\cdot 10^{-9}/f)(1.66\cdot 10^{12})(1.25\cdot 10^{13}/s^2) = (2.26\cdot 10^{18}/s^2)/f. \qquad (3.11)$$

The corresponding power spectral density $S_{\delta I}(f)$ of electrical current fluctuations $\delta I$, and rms noise current $I_n$ are

$$S_{\delta I}(f) = (1.6\cdot 10^{-19}C)^2(4.7\cdot 10^{16}/s^2)/f = 5.73\cdot 10^{-20}A^2/f \qquad \text{and} \qquad I_n = 2.4\cdot 10^{-10}\ A/f^{1/2}. \qquad (3.12)$$

### 3.2. Comparison with the experiment

Finally, an experimental value for the current noise ($I_{noise}$) is obtained by means of the detectivity and responsivity data reported by Jutao Jiang. The NEP=$P_{equiv}$ is the optical power producing a signal equal with the rms noise current. The detectivity ($D^*$) is proportional with the inverse of the noise equivalent optical power NEP=$P_{equiv}$,

$$D^* = \frac{1}{P_{equiv}} \qquad (3.13)$$

where we have assumed unit area of the detector and a detector electrical bandwidth of 1 Hz. On the other hand, the responsivity (R) is defined as the current generated per optical power unit, and therefore

$$R = \frac{I_{noise}}{P_{equiv}} \qquad (3.14)$$

This implies that the current noise can be written

$$I_{noise} = \frac{R}{D^*} \quad \text{or, equivalently,} \qquad \log I_{noise} = \log R - \log D^* \qquad (3.15)$$

The data gives the following relations for detectivity and responsivity, both as functions of the bias voltage

$$\log D^* = 9 - \frac{V}{2volts} \qquad (3.16)$$

$$\log R_{peak} = -2 + \frac{3V}{4volts} \qquad (3.17)$$

which can easily be compared with Eq. (3.12):

$$\log I_{noise} = \log R - \log D^* = \left(-2 + \frac{3V}{4volts}\right) - \left(9 - \frac{V}{2volts}\right) = -11 + \frac{5V}{4volts} \qquad (3.18)$$

For V=2 volts this experimental result is within one order of magnitude and just slightly larger than our theoretical result of $2.4\cdot 10^{-10}$ $A/f^{1/2}$obtained in Eq. (3.12) above from first principles, without any adjustable parameters. Furthermore, the observed dependence on the voltage in (3.18) is proportional to $V^{1.25}$ in the rms noise current, i.e., proportional to $V^{2.5}$ in the spectral density of current fluctuations. In the Q1/f theory result (3.8) above, the last 3 factors in the spectral density of current fluctuations are proportional to $V^{1/2}$, V, and V respectively, resulting in a total proportionality to $V^{2.5}$ for V not too small, in nice agreement with the experiment.

Often the collisions will be important. To investigate the collision-dominated case, we will consider the drift velocity of the charge carriers to be $v_d = \mu E$, where $\mu$ is the mobility of the charge carrier. The spectral density is given by

$$S_{\frac{\delta\rho}{\rho}} = S_{\frac{\delta J_{dark}}{J_{dark}}} = \frac{4\alpha}{3\pi fN}\left(\frac{eEL_p + W_0}{m^*c^2}\right) = j^{-2}S_{\delta J_{dark}}(f), \qquad (3.19)$$

where total dark current is $J_{dark} = n_oAv_d = n_oA\,\mu E$. The number of charge carriers N can be written as $n_oAL$ where $n_o$ is the doping level of the well. L is the total length of the device and $L_p$ is the length of the periodicity, or the length of the well and barrier. The effective mass, $m^*$, is related to the mass of the free particle by $m^* = 0.077m_0$. The spectral density is then

$$S_{\frac{\delta J_{dark}}{J_{dark}}} = \frac{4\alpha}{3\pi fn_0AL}\left(\frac{eEL_p + W_0}{m^*c^2}\right). \qquad (3.20)$$

This yields results similar to Eq. (3.12).

### 3.3. Conclusions

From Eqs. (3.18), (3.12), and (3.8) we see that the quantum 1/f formulas correctly describe the 1/f noise in the dark current of QWIPs, both in magnitude and in its dependence on bias. The photo-generation process does not[31] have quantum 1/f noise. These new analytical formulas allow for the first time optimization of the QWIPs with respect to parameters such as choice of the semiconductors used, i.e., the effective mass of the carriers, band and subband parameters, well geometry, etc. Quantum well intersubband photodetectors (QWIPs) can be extended in principle from infrared into the THz region[32,33].

The authors thankfully acknowledge support by Dwight Woolard through the US Army research Office.

**REFERENCES**


1. A. van der Ziel, "*Noise in Solid State Devices and Circuits*", Wiley-Interscience, New York, 1986, Ch. 11.
2. P.H. Handel: "1/f Noise - an 'Infrared' Phenomenon", Phys. Rev. Letters 34, 1492-1495 (1975).
3. P.H. Handel: "Nature of 1/f Phase Noise", Phys. Rev. Letters 34, 1495-1498 (1975).
4. P.H. Handel: "Quantum Approach to 1/f Noise", Phys. Rev. 22A, 745-757 (1980).
5. T. Sherif and P.H. Handel: "Unified Treatment of Diffraction and 1/f Noise", Phys. Rev. A 26, 596-602 (1982).
6. P.H. Handel and D. Wolf: "Characteristic Functional of Quantum 1/f Noise", Phys. Rev. A26, 3727-30 (1982).
7. G.S. Kousik, C.M. van Vliet, G. Bosman and P.H. Handel: "Quantum 1/f Noise Associated with Ionized Impurity Scattering and Electron-phonon Scattering in Condensed Matter", Advances in Physics 34, 663-702 (1985).
8. P.H. Handel: "Infrared Divergences, Radiative Corrections and Bremsstrahlung in the Presence of a Thermal Equilibrium Radiation Background", Phys. Rev. A38,.3082-3085 (1988).
9. P. H. Handel, The quantum 1/f effect and the general nature of 1/f noise, *Archiv für Elektronik und Übertragungstechnik (AEÜ)* **43**, 261-270 (1989).
10. A. van der Ziel, Unified presentation of 1/f noise in electronic devices: fundamental 1/f noise sources, *Proc. IEEE* **76**, 233-258 (1988; review paper); The experimental verification of Handel's expressions for the Hooge parameter, *Solid-State Electronics* **31**, 1205-1209 (1988); Semiclassical derivation of Handel's expression for the Hooge parameter, *J. Appl. Phys*. **63**, 2456-2455 (1988); **64**, 903 (1988).
11. A. van der Ziel et. al., Extensions of Handel's 1/f Noise Equations and their Semiclassical Theory, *Phys. Rev. B* 40, 1806-1809 (1989).
12. P. H. Handel, Fundamental quantum 1/f noise in small semiconductor devices", *IEEE Trans. on Electr. Devices* **41**, 2023-2033 (1994).
13. P. H. Handel, Nature of 1/f frequency fluctuation in quartz crystal resonators, *Solid-State Electronics* **22**, 875-876 (1979).
14. P. H. Handel, Coherent and Conventional Quantum 1/f Effect, *Physica Status Solidi b* **194,** 393-409 (1996).
15. P. H. Handel, Noise, low frequency, *Wiley Encyclopedia of Electrical and Electronics Engineering*, vol. 14, pp. 428-449, John Wiley & Sons, Inc., 1999.
16. P. H. Handel, Quantum 1/f optimization of quantum sensing in spintronic, electro-optic and nano-devices, *Proc. of SPIE conf. on "Quantum Sensing: Evolution and Revolution from Past to Future,"* San Jose, CA, Jan. 27-30, 2003. (Invited Paper), vol. 4999, Editors M. Razeghi and G. J. Brown, pp 322-336, (July 2003).
17. T.F. George and Peter H. Handel "Quantum 1/f Effect Based on Quantum Information Theory" Proc. *Int. Conf. on Squeezed States and Uncertainty Relations* (ICSSUR) Besancon 2005, p. 152 (invited paper)
18. P. H. Handel and A. G. Tournier, Nanaoscale engineering for reducing phase noise in electronic devices, *Proc. IEEE* **93**, 1784-1814 (2005).
19. P.H. Handel, C.M. Van Vliet and A. van der Ziel: "Derivation of the Nyquist-1/f Noise Theorem", Proc. VII Int. Conf. on Noise in Physical Systems and III Int. Conf. on 1/f Noise, Montpellier, May 17-20, 1983, V.M. Savelli, G. Lecoy and J.P. Nougier Editors, North-Holland Publ. Co. (1983), p. 93-96.
20. P.H. Handel: "Any Particle Represented by a Coherent State Exhibits 1/f Noise", Ibid., p. 97-100.
21. See P.H. Handel, "General Quantum 1/f Bibliography", kept current at the Physics Dept., Univ. of Missouri, St. Louis MO 63121.
22. C.M. Van Vliet: "Quantum Electrodynamical Theory of Infrared Effects in Condensed Matter. II.: Radiation Corrections of Cross Sections and Scattering Rates and Quantum 1/f Noise." Physica A, 165, 126-155 (1990).
23. P.H. Handel: "The Nature of Fundamental 1/f Noise", *XII. Internatl. Conf. on Noise in Physical Systems and 1/f Fluctuations*, St. Louis, MO, Aug. 16-20, 1993, AIP Conf. Proceedings No. 285, P.H. Handel and A.L. Chung, Editors, pp. 162-171 (1993).
24. A. Balandin, K.L. Wang, R. Li, A. Svizhenko, S. Bandyopadhyay: "The Fundamental 1/f Noise and the Hooge Parameter in Semiconductor Quantum Wires", IEEE Trans. on Electron Devices, 46, 1240-4 (1999).
25. Tacano, M.: "Hooge fluctuation parameter of semiconductor microstructures." IEEE Transactions on Electron Devices 40, 2060-4 (1993).
26. M. Tacano, A. Ando, I. Shibasaki, S. Hashiguchi, J. Sikula and T. Matsui: "Dependence of Hooge Parameter of Compound Semiconductors", AIP Proc. #466, P.H. Handel and A.L. Chung Editors, (1999), pp. 35-41.
27. A. Balandin, K.L. Wang, A. Svizhenko and S. Bandyopadadhyay: "The Effects of Low-Dimensionality on the Quantum 1/f Noise", AIP Proc. # 466, P.H. Handel and A.L. Chung Editors, (1999), pp. 131-134.

28. P.H. Handel, "Coherent States Quantum 1/f Noise and the Quantum 1/f Effect" in *"Noise in Physical Systems and 1/f Noise"* (Proceedings of the VIIIth International Conference on Noise in Physical Systems and 1/f Noise) edited by A. D'Amico and P. Mazzetti, Elsevier, New York, 1986, p. 469.
29. J. Jiang, Infrared focal plane array based on GaInAs/InP quantum well infrared photodetectors, Northwestern Univ. Doctoral Thesis, Evanston, IL, 2003 (Adviser: Manijeh Razeghi).
30. P. H. Handel, Quantum 1/f optimization of quantum sensing in spintronic, electro-optic and nano-devices, *Proc. of SPIE conf. on "Quantum Sensing: Evolution and Revolution from Past to Future,"* San Jose, CA, Jan. 27-30, 2003. (Invited Paper), vol. 4999, Editors M. Razeghi and G. J. Brown, pp 322-336, (July 2003).
31. P. H. Handel, Absence of Quantum 1/f Noise in the Photoelectric Current of Junction or MIS Photodetectors, *IEEE Transactions on Electron Devices* **40**, 833 (1993).
32. P. Harrison, M.A. Gadir, N.E.I. Etteh and R.A. Soref, The Physics of THz QWIPs, Proc. Ninth Int. Conf. on THz Electronics, (2000).
33. D. L. Woolard, E. R. Brown, M. Pepper and M. Kemp, THz frequency sensing and imaging: A time of reckoning future applications?" *Proc. IEEE* **93**, 1722-1743 (2005).