QUANTUM 1/f NOISE IN THZ DETECTORS AND GENERATORS

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1/f noise limits the performance of many sensors and detectors, also in the THz spectral region. Quantum 1/f noise is a new aspect of quantum mechanics, arising from the conventional and coherent quantum 1/f effects. The quantum 1/f theory is introduced and used to obtain analytical expressions of noise in n^+p diodes, Schottky diode THz detectors and mixers, a new type of suggested MEMS resonator based Thz detectors, QWIPs and bolometers. For the DAR and for THz communications, where frequency resolution is important, we also consider phase noise in RTDs and GaN/AlGaN HFET-based THz generators. In n+p diodes and Schottky diodes the results obtained show inverse proportionality of 1/f noise with the life time of the carriers. The suggested MEMS-based THz detectors can have quantum 1/f-phase noise-limited detectivities above $2.!0^{11}Hz^{1/2}/W$. Quantum 1/f responsivity and detectivity formulas are obtained for the first time in the 1/f region here for QWIPS, and applied to the THz region. For RTDs analytical expressions for the phase noise and frequency fluctuations are obtained.

Keywords: THz Radiation, 1/f Noise, RTDs, QWIPS, Quantum 1/f noise, THz detectors, quantum well devices, MEMS resonators.

1. Introduction

THz radiation is of increasing importance for detection of bacteriological and chemical agents, including imaging and remote detection with differential absorption radar (DAR). The THz generator and detector used are based on devices with fluctuations[1-6] that reduce the detectivity, the spectral resolution, and the useful range of the DAR[7]. Less understood, and more difficult to control is the 1/f noise and its resulting phase noise.

Fluctuations with a spectral density proportional to 1/f are found in a large number of systems in science, technology and everyday life. These fluctuations are known as 1/f noise in general. They have first been noticed by Johnson in early amplifiers, have limited the performance of vacuum tubes in the thirties and forties, and have later hampered the introduction of semiconductor devices.

The present paper is focused on reducing the 1/f noise in THz hardware. It starts with a *special case* of the general 1/f noise phenomenon, the Quantum 1/f Effect with its conventional and coherent contributions[1-6]. The 1/f fluctuations are a necessary consequence of the mathematical homogeneity of the dynamical (or physical) equations describing the motion of an arbitrary chaotic or stochastic nonlinear system. A sufficient criterion was derived by the author[6]. It indicates if an arbitrary system governed by a given system of nonlinear integro-differential equations will exhibit 1/f noise. The criterion was applied to several particular systems, and is used to predict the fundamental quantum 1/f effect as a *special case*.

2. Conventional Quantum 1/f Effect

This effect[1-6] is present in any cross section or process rate involving charged particles or current carriers. The physical origin of quantum 1/f noise is easy to understand. Consider for example Coulomb scattering of current carriers, e.g., electrons on a center of force. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by DeBroglie waves of a frequency corresponding to their energy. However, some of the electrons have lost energy in the scattering process, due to the emission of bremsstrahlung. Therefore, part of the outgoing DeBroglie waves is shifted to slightly lower frequencies. When we calculate the probability density in the scattered beam, we obtain also cross terms, linear both in the part scattered with and without bremsstrahlung. These cross terms oscillate with the same frequency as the frequency of the emitted bremsstrahlung photons. The emission of photons at all frequencies results therefore in probability density fluctuations at all frequencies. The corresponding current density fluctuations are obtained by multiplying the probability density fluctuations by the velocity of the scattered current carriers. Finally, these current fluctuations present in the scattered beam will be noticed at the detector as low frequency current fluctuations, and will be interpreted as fundamental cross section fluctuations in the scattering cross section of the scatterer. While incoming carriers may have been Poisson distributed, the scattered beam will exhibit super-Poissonian statistics, or bunching, due to this new effect which we may call quantum 1/f effect. The quantum 1/f effect is thus a many-body or collective effect, at least a two-particle effect, best described through the two-particle wave function and two-particle correlation function.

Let us estimate the magnitude of the quantum 1/f effect semiclassically by starting with the classical (Larmor) formula $2q^2\mathbf{a}^2/3c^3$ for the power radiated by a particle of charge q and acceleration $\mathbf{a}$. The acceleration can be approximated by a delta function $\mathbf{a}(t) = \Delta\mathbf{v}\delta(t)$ whose Fourier transform $\Delta\mathbf{v}$ is constant and is the change in the velocity vector of the particle during the almost instantaneous scattering process. The one-sided spectral density of the emitted bremsstrahlung power $4q^2(\Delta\mathbf{v})^2/3c^3$ is therefore also constant. The number $4q^2(\Delta\mathbf{v})^2/3hfc^3$ of emitted photons per unit frequency interval is obtained by dividing with the energy hf of one photon. The probability amplitude of photon emission $[4q(\Delta\mathbf{v})^2/3hfc^3]^{1/2}e^{i\gamma}$ is given by the square root of this photon number spectrum, including also a phase factor $e^{i\gamma}$. Let ψ be a representative Schrödinger catalogue wave function of the scattered outgoing charged particles, which is a single-particle function, normalized to the actual scattered particle concentration. The beat term in the probability density $\rho=|\psi|^2$ is linear both in this bremsstrahlung amplitude and in the non-bremsstrahlung amplitude. Its spectral density will therefore be given by the product of the squared probability amplitude of photon emission (proportional to 1/f) with the squared non-bremsstrahlung amplitude which is independent of f. The resulting spectral density of fractional probability density fluctuations is obtained by dividing with $|\psi|^4$ and is therefore

$$|\psi|^{-4}S_{|\psi|^2}(f) = 8q^2(\Delta\mathbf{v})^2/3hfNc^3 = 2\alpha A/fN = j^{-2}S_j(f), \qquad (1)$$

where $\alpha = e^2/\hbar c = 1/137$ is the fine structure constant and $\alpha A = 4q^2(\Delta v)^2/3hc^3$ is known as the infrared exponent in quantum field theory, and is known as quantum 1/f noise coefficient, or Hooge constant, in electrophysics.

The spectral density of current density fluctuations is obtained by multiplying the probability density fluctuation spectrum with the squared velocity of the outgoing particles. When we calculate the spectral density of fractional fluctuations in the scattered current j, the outgoing velocity simplifies, and therefore Eq. (1) also gives the spectrum of current fluctuations $S_j(f)$, as indicated above. The quantum 1/f noise contribution of each carrier is independent, and therefore the quantum 1/f noise from N carriers is N times larger; however, the current j will also be N times larger, and therefore in Eq. (1) a factor N was included in the denominator for the case in which the cross section fluctuation is observed on N carriers simultaneously.

The fundamental fluctuations of cross sections and process rates are reflected in various kinetic coefficients in condensed matter, such as the mobility μ and the diffusion constant D, the surface and bulk recombination speeds s, and recombination times τ, the rate of tunneling j_t and the thermal diffusivity in semiconductors. Therefore, the spectral density of fractional fluctuations in all these coefficients is given also by Eq. (1).

When we apply Eq. (1) to a certain device, we first need to find out which are the cross sections σ or process rates which limit the current I through the device, or which determine any other device parameter P, and then we have to determine both the velocity change $\Delta\mathbf{v}$ of the scattered carriers and the number N of carriers simultaneously used to test each of these cross sections or rates. Then Eq. (1) provides the spectral density of quantum 1/f cross section or rate fluctuations. These spectral densities are multiplied by the squared partial derivative $(\partial I/\partial\sigma)^2$ of the current, or of the device parameter P of interest, to obtain the spectral density of fractional device noise contributions from the cross sections and rates considered. After doing this with all cross sections and process rates, we add the results and bring (factor out) the fine structure constant α as a common factor in front. This yields excellent agreement with the experiment in a large variety of samples, devices and physical systems.

Eq. (1) was derived in second quantization, using the commutation rules for boson field operators. For fermions one repeats the calculation replacing in the derivation the commutators of field operators by anticommutators, which yields[1-6]

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/f(N-1) \qquad (2)$$

This causes no difficulties, since $N \geq 2$ for particle correlations to be defined, and is practically the same as Eq. (1), since usually $N >> 1$. Eqs. (1) and (2) suggest a new notion of physical cross sections and process rates which contain 1/f noise, and express a fundamental law of physics, important in most high-technology applications.

We turn now to the connection to the coherent Quantum 1/f Effect, essentially caused by the uncertainty of the electron mass, by the coherent state of the field of the electron. The coherent state has an uncertain energy. The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy E_m, per unit length, of the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int(B^2/8\pi)d^3x = [nevS/c]^2 \ln(R/r) \qquad (3)$$

is much smaller than the total kinetic energy E_k of the drift motion of the individual carriers

$$E_k = \Sigma_i m_i v^2/2 = nSmv^2/2 = E_m/s. \qquad (4)$$

Here we have introduced the magnetic field B, the carrier concentration n, the cross sectional area S and radius r of the cylindrical sample (e.g., a current carrying wire), the radius R of the electric circuit, and the "coherece ratio"

$$s=E_m/E_k =2ne^2S/mc^2\ln(R/r) =2e^2N'/mc^2, \qquad (5)$$

where N' =nS is the number of carriers per unit length of the sample and the natural logarithm ln(R/r) has been approximated by one in the last form. We expect the observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2)S_\mu(f)=[1/(1+s)][2\alpha A/fN]+[s/(1+s)][2\alpha/\pi fN] \qquad (6)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample and the frequency f are brought to the numerator of the left hand side. In this equation $\alpha A=2\alpha(\Delta\mathbf{v}/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum 1/f effect[1-6]. The second term on the right hand side brings the coherent quantum 1/f effect contribution $2\alpha/\pi fN$, caused by the uncertainty in the energy of a charged particle of a given momentum. This uncertainty is caused by the coherent state of the field of the charged particle, which is not an energy eigenstate. Equation (6) is limited to quantum 1/f mobility (or diffusion) fluctuations, and does not include the quantum 1/f noise in the surface and bulk recombination cross sections, in the surface and bulk trapping centers, in tunneling and injection processes, in emission or in transitions between two solids.

Note that the coherence ratio s introduced here is one for the critical value N' = N" = $2{\cdot}10^{12}$/cm., e.g. for a cross section S = $2{\cdot}10^{-4}$ cm^2 of the sample when n = 10^{16}. For small samples with N'<<N" only the first term survives, while for N'>N" the second term in Eq. (6) is dominant.

3. Derivation of Quantum 1/f Noise in n^+-p Diodes

For a diffusion limited n^+-p junction the current is controlled by diffusion of electrons into the p - region over a distance of the order of the diffusion length $L = (D_n\tau_n)^{1/2}$ which is shorter than the length w_p of the p - region in the case of a long diode. If N(x) is the number of electrons per unit length and D_n their diffusion constant, the electron current at x is

$$I_{nd} = - eD_n dN/dx, \qquad (7)$$

where we have assumed a planar junction and taken the origin x = 0 in the junction plane. Diffusion constant fluctuations, given by kT/e times the mobility fluctuations, will lead to local current fluctuations in the interval Δx

$$\delta\Delta I_{nd}(x,t) = I_{nd}\Delta x\delta D_n(x,t)/D_n. \qquad (8)$$

The normalized weight with which these local fluctuations representative of the interval Δx contribute to the total current I_d through the diode at x = 0 is determined by the appropriate Green function and can be shown to be (1/L)exp(-x/L) for $w_p/L >> 1$. Therefore the contribution of the section Δx is

$$\delta\Delta I_d(x,t) =(\Delta x/L)\exp(-x/L)I_{nd}\delta D_n(x,t)/D_n, \qquad (9)$$

with the spectral density

$$S_{\Delta I_d}(x,f) =(\Delta x/L)^2 \exp(-2x/L)\, I_{nd}^2\, S_{D_n}(x,f)/D_n^2. \qquad (10)$$

For mobility and diffusion fluctuations the fractional spectral density is given by $\alpha_{Hnd}/fN\Delta x$, where α_{Hnd} is determined from quantum 1/f theory according to Sec 2. With Eq. (7) we obtain then

$$S_{\Delta I_d}(x,f) =(\Delta x/L^2)\exp(-2x/L)(eD_n dN/dx)^2\alpha_{Hnd}/fN. \qquad (11)$$

The electrons are distributed according to the solution of the diffusion equation, i.e.

$$N(x) = [N(0) - N_p] \exp(-x/L); \qquad dN/dx = -\{[N(0) - N_p]/L\} \exp(-x/L). \quad (12)$$

Substituting into Eq. (11) and simply summing over the uncorrelated contributions of all intervals Δx, we obtain

$$S_{Id}(f) = \alpha_{Hnd}(eD_n/L^2)^2 \int[N_0-N_p]^2e^{-4x/L}dx/\{[N(0) -N_p]\, e^{-x/L} + N_p\}. \quad (13)$$

We note that $eD_n/L^2 = e/\tau_n$. With the expression of the saturation current $I_0 = e(D_n/\tau_n)^{1/2}N_p$ and of the current $I = I_0[\exp(eV/kT) - 1]$, we can carry out the integration

$$S_{Id}(f) = \alpha_{Hnd}(eI/f\tau_n) \int du/(au + 1) = \alpha_{Hnd}(eI/f\tau_n)\, F(a). \qquad (14)$$

Here we have introduced the notations

$$u = \exp(-x/L), \quad a = \exp(eV/kT) - 1, \quad F(a) = 1/3 - 1/2a + 1/a^2 - (1/a^3)\ln(1+a). \quad (15)$$

Eq. (14) gives the diffusion noise as a function of the quantum 1/f noise parameter αH_{nd}. A similar result can be derived for the quantum 1/f fluctuations of the recombination rate r in the bulk of the p - region, the only difference being the presence of α_{Hnr} instead of α_{Hnd} in Eq. (14). The total noise is the given by Eq. (14) with α_{Hnd} replaced by the sum $\alpha_{Hnd} + \alpha_{Hnr}$

$$S_{Id}(f) = (\alpha_{Hnd} + \alpha_{Hnr})(eI/f\tau_n)F(a). \qquad (16)$$

4. Schottky Diode Detectors

The development of spectral-based early warning sensor capabilities is critically dependent[7] on ultra-low noise sensors, detectors and UHF electronic components, e.g., Schottky diode rectifiers, multipliers, mixers, and heterodyne receivers. Schottky barrier diodes are classical room temperature detectors that respond directly with a DC current or voltage, proportional to the square of the a.c. input voltage amplitude. They are particularly useful in the THz frequency domain, but can be extended into the infrared region. They consist of a rectifying metal contact on a semiconductor sample, or of a semimetal-semiconductor heterojunction, as recently demonstrated[8]. On the other hand, a new understanding of the fundamental 1/f noise has emerged from the quantum 1/f theory, providing analytical expressions, and optimization methods for all sensors, detectors, amplifiers, resonators, and oscillators. The present contribution is applying these new methods to various Schottky diodes, for a more complete, first principles, derivation of their Noise Equivalent Power (NEP), which naturally includes a 1/f noise caused part. They are also applied here to other sensor methodologies, technologies and instrumentation that are vital for the most challenging bio-chemical and radiation agent detection and identification systems.

Schottky barrier diodes have a characteristic

$$I=I_o[\exp(eV/AkT)-1]. \qquad (17)$$

Here I_o is the electron current flowing in both directions at zero bias. This yields a response I_d and a NEP

$$I_d=P_a[dI^2/dV^2][2dI/dV]^{-1};\ NEP=[2dI/dV][d^2I/dV^2]^{-1}[S_I(f)\Delta f]^{1/2}=(2AkT/e)\,[S_I(f)\Delta f]^{1/2} \quad (18)$$

where P_a is the available power of the signal source or antenna assumed to be matched to the detector, R=2kT/e is the responsivity, 1<A<2 the ideality, and $S_I(f)$ is the power spectral density of noise

$$S_I(f)= 2e(2I_o+I_d) +(e^3\alpha_H I_o/3kTf)[v_r v_d/(v_r+v_d)][2N_d(V_{dif}-V)/e\varepsilon]^{1/2}. \qquad (19)$$

This expression takes into account the image force and is valid both in the diffusion-limited (v_r>>v_d) and thermionic-limited (v_d>>v_r) limiting regimes, and in between. Here V_{dif}-V is the Schottky barrier height, Δf the bandwidth, N_d the concentration of donors, e the elementary charge, α_H the Hooge constant, and $\varepsilon=\varepsilon_r\varepsilon_o$ the permittivity of the semiconductor. The first term in Eq. (19) is the (white) shot noise. For a small, heavily doped semiconductor, our conventional collisional quantum 1/f expression of α_H is

$$\alpha_H=(4\alpha/3\pi)(6kT/m^*c^2)=10^{-9}Tm/(4m^*100K), \qquad (20)$$

with $v_r=(kT/2\pi m^*)^{1/2}$, $v_d=\mu[2eN_d(V_{dif}-V)/\varepsilon]^{1/2}$. For larger semiconductor contacts the capacitance of the diode increases, and a coherent quantum 1/f contribution, shown e.g. in Eq, (2.30) of our reference[2], may yield a much larger α_H. Here α is the fine structure constant 1/137, m the electron's mass and m* its effective mass. Substituting Eq. (20) into Eq. (19), we obtain

$$S_I(f)=2e(2I_o+I_d)+(8\alpha Ie^3/3pm^*c^2f)[v_r v_d/(v_r+v_d)][2N_d(V_{dif}-V)/e\varepsilon]^{1/2}. \qquad (19')$$

We notice the absence of any direct temperature dependence. Quantum 1/f noise is a property of physical cross sections, a new aspect of quantum mechanics.

4.1. *Schottky diodes as THz mixers*

As a THz wave mixer, with g_o=dI/dV, this diode would have quantum 1/f phase noise[9] and available matched gain

$$G_{av}=(P_{av})_{out}/P_a=R_aV_p[d^2I/dV^2]^2[4dI/dV]^{-1}\{[1+g_i(R_a+r)]^2+\omega^2C^2(R_a+r)]^2\}^{-1}, \quad (21)$$

and, with Q1/f noise $[2n\pi fQ]^{-1}$ in the radiation resistance, an equivalent available noise power $(P_{eq})_a$ per Hz at the antenna of quality factor Q

$$(P_{eq})_a=(n_o+F_2-1)\{[kT+S_I(f)]/G_{av}\} +1/[2n\pi fQ]; \quad n=W/\omega\hbar \qquad (22)$$

with an addition due to the phase noise from the mixer. Here R_a is the source (or antenna) resistance, V_p the pump voltage amplitude, r is the series resistance of the bulk region of the diode, C and g_i the capacity and conductance in parallel characterizing the space charge region, F_2 the noise figure of the IF amplifier, $n_o=eI_d/2kTg_o$, and ω the THz source frequency. We have denoted by W the antenna oscillation energy, and n the number of oscillator quanta corresponding to W.

We shall give below more examples of application of the quantum 1/f noise and phase noise theory to THz spectral-based sensing hardware.

5. MEMS Resonator THz Detectors

Pairs of identical MEMS resonators could be used as THz wave detectors instead of bolometers. Indeed, pairs of identical MEMS resonators with identical excitations can be made to have a very small difference in their resonance frequencies. The corresponding beat frequency will therefore fluctuate if there is no coupling between them, and will tend to be zero due to spontaneous synchronization, when there is even a weak coupling present. An increasing temperature difference between the resonators, caused by the absorption of a THz radiation signal will result in an increasing beat frequency that can be amplified and registered acoustically and electrically.
Any temperature variations cause thermal dilatation and changes in the elastic constants and the speed of sound. These, in turn, affect the resonance frequency through the temperature coefficient $\alpha_T = d\nu/\nu dT$ of the resonance frequency ν. For silicon MEMS resonators, Nguyen[10] reported $\alpha_T = -10^{-5}$/K. Denoting by G the thermal exchange rate in w/K, we obtain for a temperature fluctuation δT in a MEMS resonator of heat capacity C in the presence of an incident THz radiation power P

$$Cd\delta T/dt = -G\delta T + \eta P. \tag{23}$$

Here η is the fraction of the incident radiation that is effectively absorbed in the MEMS resonator. In stationary conditions, with constant P we obtain the response

$$\delta T = \eta P/G = \delta\nu/\alpha_t\nu, \tag{24}$$

where we have also displayed the fractional frequency change $\delta\nu/\nu$ of the resonator along with the temperature coefficient α_t. It could be made as low as 10^{-5}/K when high stability of the oscillator would be desired, as reported by Nguyen[10]. Here larger values, e.g., 10^{-3}/K will be used. The expected responsivity in Hz/w is thus

$$R = \delta\nu/P = \eta\nu\alpha_t/G. \tag{25}$$

For instance G =4.8 10^{-6}w/K for $\nu=10^9$Hz as inferred from Table I in Vig and Kim [11]. With η =0.9 and $\alpha_T = -10^{-3}$/K as mentioned above, we obtain R =2 10^{11}Hz/w. The resulting beat frequency $\delta\nu$ can be measured and displayed. For P given by fractions of a microwatt, it is in the audible domain, and can be used as an alert.

Temperature fluctuations δT are particularly important in small resonators, such as MEMS resonators, and will reduce the detectivity D*, defined as 1/NEP, the reciprocal of the noise equivalent power. In thermal equilibrium, from thermodynamics,

$$\langle(\delta T)^2\rangle = kT^2/C = kT^2/c_vV, \tag{26}$$

where k is Boltzmann's constant and C the heat capacity of the resonating body, i.e., volumetric specific heat c_v, times volume V. From the exponential decay implied in Eq. (23) above for P=0, we obtain

$$\delta T(t) = \delta T(0)\exp(-Gt/C);\ \ \langle\delta T(t)\delta T(t+\tau)\rangle = \langle(\delta T)^2\rangle\exp(-Gt/C). \tag{27}$$

Therefore, using the Wiener Khintchine theorem, the spectral densities of temperature fluctuations and of relative frequency fluctuations $y=\delta\nu/\nu$ are

$$<(\delta T)^2>_f \equiv S_{\delta T}(f) = (4kT^2/G)/[\,1 + (2\pi fC/G)^2], \qquad (28)$$

$$<(\delta\nu/\nu)^2>_f \equiv S_{\delta\nu/\nu}(f) = (4\alpha_T{}^2kT^2/G)/[\,1 + (2\pi fC/G)^2], \qquad (29)$$

With the data used above we obtain for the expected thermal $\delta\nu/\nu$ the estimate $5\,10^{-10}(Hz)^{-1/2}$. In the case of MEMS resonators with frequencies from 100 kHz to 10 GHz, these results have been graphed by Vig and Kim [11] for heat exchange G caused by radiation, and by both radiation and conduction. Conduction turns out to be dominant in MEMS and NEMS with G between 1 and $5{\cdot}10^{-6}$w/k, causing $\tau=C/G$ to be of the order of 10^{-5}s and smaller, so the second term in the denominator can be neglected.

Note that the heat conduction rate G to the support heat sink is proportional to the thickness z and width W, while inversely proportional to the length L of the resonating MEMS bar. Therefore, for self-similar scaling of the size and volume, the frequency noise spectrum will be proportional with the -1/3 power of the volume V, and the proportionality with ν^2 may introduce an additional dependence of phase noise, roughly with $V^{-2/3}$ when the same resonant mode is used. In conclusion, an inverse proportionality of classical $1/f^2$ phase noise $\mathcal{L}(f)$ caused by temperature fluctuations on the resonant volume V can be expected as a first rough approximation

$$\mathcal{L}(f) = (\nu^2/2f^2)S_{\delta\nu/\nu}(f) = (\nu^2/2f^2)(4\alpha_T{}^2kT^2/G) \approx \mathrm{const}/V. \qquad (30)$$

The total frequency fluctuations are obtained by adding the quantum 1/f frequency fluctuations and the classical fluctuations unrelated to temperature fluctuations to Eq. (29)

$$<(\delta\nu/\nu)^2>_f \equiv S_{\delta\nu/\nu}(f) = (\beta/fQ^4)/[1/V + V/\varepsilon^6] + (4\alpha_T{}^2kT^2/G)/[1 + (2\pi fC/G)^2] \\ + (kT/Q_l{}^2P) + \Gamma^2<(\delta a)^2>_f \qquad (31)$$

On the r.h.s. the first term is the quantum 1/f noise[9], the third is the Johnson noise [9] and the last is the acceleration noise[3], if present, e.g., for MEMS resonators THz detectors mounted on a mobile platform, e.g., an airplane. Johnson noise is the thermal noise from the mechanical dissipation, or mechanical resistance. All are derived in reference[9]. Here we calculate the NEP. We just solve Eq. (24) for P and replace $\delta\nu/\nu$ equal with the square root of Eq. (31)

$$\mathrm{NEP} = 1/D^* = (G/\alpha_t\eta)\delta\nu/\nu = (G/\alpha_t\eta)\{(\beta/fQ^4)/[1/V + V/\varepsilon^6] \\ + (4\alpha_T{}^2kT^2/G)/[1 + (2\pi fC/G)^2] + (kT/Q_l{}^2P) + \Gamma^2<(\delta a)^2>_f\}^{1/2}. \qquad (32)$$

The corresponding detectivity D* is expressed here in $(Hz)^{1/2}$/w. Assuming a realistic value of $S_{\delta\nu/\nu}(f) = 10^{-18}/f$, we obtain $\delta\nu/\nu=10^{-9}/Hz^{1/2}$ at 1 Hz., and with the data used above, $D^* = 2{\cdot}10^{11}Hz^{1/2}/w$, increasing even more for larger bandwidth, as long as the 1/f dependence holds.

We conclude that this is a competitive detectivity for MEMS resonator THz detectors, and it does not require cooling. Therefore, we recommend considering this alternative.

6. Quantum Well Intersubband Photodetectors

Quantum well intersubband photodetectors (QWIPs) could be used [12] in the THz frequency domain, if properly designed and optimized. However, they yield a dark current that is always present, even without illumination and background radiation. Fluctuations of this dark current will compete with the signal to be detected and limit the detectivity. A first calculation of fundamental 1/f noise in the QWIPS dark current, based on the universal quantum 1/f formulas, is presented and compared with experimental data on GaInAs/InP QWIPs.

6.1. *Introduction to QWIPs*

The Quantum Well Intersubband Photodetector is a semiconductor photon detector that responds to absorption within its bandgaps. The structure is designed to allow an excited carrier within the well to escape, joining others to form what is called the dark current. Because the dark current is a factor in optimizing device performance, we will investigate the noise associated with this current. The noise is described by the conventional quantum 1/f effect [4], [6], [13] which describes the fundamental fluctuations in the physical cross sections and process rates that generate the dark current. The calculation of the 1/f noise is then compared to experimental data from Jutao Jiang (and Manijeh Razeghi) [14]. A similar calculation was performed earlier for n+p junction infrared detectors and MIS infrared detectors [15].

According to the quantum 1/f noise theory, [4], [6], [13], due to minute bremsstrahlung energy losses in scattering processes, the scattered current will exhibit fractional quantum fluctuations with the same (actually twice the ..) spectral density as the number of the emitted bremsstrahlung photons. The later is known to be inversely proportional to the frequency. The physical scattering cross section σ (and in general, the rate Γ of any other quantum process) is therefore observed to show fundamental quantum fluctuations with a 1/f spectral density of fractional fluctuations given by

$$S_{\delta\sigma/\sigma}(f) = S_{\delta\Gamma/\Gamma}(f) = S_{\delta j/j}(f) = (4\alpha/3\pi fN)(\mathbf{\Delta v}/c)^2, \qquad (33)$$

where $\alpha = (1/4\pi\varepsilon_o)e^2/\hbar c = 1/137$ is the fine structure constant, $\mathbf{\Delta v}$ is the vector velocity change of the charged particles (here electrons) in the scattering process, and N is the number of particles that defines the scattered current j, i.e., the cross section σ or process rate Γ in general. Eq. (33) can be easily derived semiclassically by dividing the bremsstrahlung power given by the dipolar radiation formula by the energy hf of a single photon

$$2\alpha/3\pi f = (1/4\pi\varepsilon_o)(2e^2\mathbf{a}_f^2/3c^3)/hf. \quad (34)$$

The subscript f means that only the contribution $\mathbf{a}_f=\mathbf{\Delta v}$ per unit frequency interval is included, at the frequency f. It is independent of f at low frequencies. The factor N in the denominator of (33) is due to the independent phase of the noise contributions from each electron.

6.2. *Theory*

Applying an electric field to the quantum well accelerates the electrons, and the resulting drift velocity gives rise to the dark current. Conservation of energy gives an expression for this velocity

$$\frac{m}{2}(\Delta\upsilon)^2 = (L_b + L_w)eE + |W_0|, \quad (35)$$

where m is the electron mass, L_b is the barrier thickness, L_w is the width of the well, E is the magnitude of the applied field and W_0 is the binding energy of the well. Here we assume that the carriers move under uniform acceleration between the wells and that the effects of collisions between the carriers are small enough to be ignored. The conventional quantum 1/f effect that occurs in these devices is the result of fluctuations in the capture cross section that determines the mobility and the dark current. According to (33), the spectral density of these current fluctuations is given by

$$S_{\frac{\delta\rho}{\rho}} = S_{\frac{\delta j_{dark}}{j_{dark}}} = \frac{4\alpha}{3\pi fN}\left(\frac{\Delta\upsilon}{c}\right)^2 = j^{-2}S_{\delta j_{dark}}(f), \quad (36)$$

where N is the number of carriers defining the current. Substituting the velocity change from Eq. (35), we obtain

$$S_{\delta j_{dark}} = \frac{4\alpha}{3\pi fN}\left(\frac{2}{mc^2}\right)\left[(L_b + L_w)eE + W_0\right] j_{dark}^2. \quad (37)$$

Next, recalling the relation between the dark particle-current j_{dark} and the number of carriers,

$$j_{dark} = \left\langle\frac{dN}{dx}\right\rangle\langle\upsilon_{carriers}\rangle. \quad (38)$$

Classically, an electron accelerated between two points has a final velocity of

$$\upsilon_{end} = \sqrt{2a(\Delta l)} \quad (39)$$

and we write the average velocity of the carriers as

$$<\upsilon_{carriers}>=\tfrac{1}{2}\sqrt{2a(\Delta l)} = \tfrac{1}{2}\sqrt{2a(L_b + L_w)}\ . \quad (40)$$

To simplify, we let $L = L_b + L_w$ represent the lattice constant of the quantum well and use $a = eE/m$ for the acceleration. The dark current is then

$$j_{dark} = \frac{N}{L}\frac{1}{2}\sqrt{\frac{2eEL}{m}} \quad (41)$$

Finally, with this relation between the number of carriers and the dark current, the spectral density can be simplified

$$S_{\delta j_{dark}} = \frac{8\alpha}{3\pi fmc^2}\left(\frac{\sqrt{eE}}{\sqrt{2Lm}}\right)\left[LeE + W_0\right] j_{dark} \tag{42}$$

To quantify the dark current fluctuations, we now apply the data and parameters of the GaInAs/InP QWIP fabricated and tested by Jutao Jiang [14], using specific structure parameters [16].

Table 1. Values of the GaInAs/InP QWIP Parameters

PARAMETER:	Value used	
Dark Current j_{dark} at 77K	2	μA
MQW lattice period L	570	Å
Total voltage applied 20EL	2	V
Voltage applied on 1 well = EL	0.1	V
Binding energy in the well W_0	0.1	eV

Next, we apply the specific parameters of the QWIP device to our derived spectral density equation (42). The binding energy, W_0, for the well is 0.251eV. The combined length of the barrier and the well is 571 angstroms. At low temperatures (around 70K), the dark current is around 2μA. Plugging these values shown in Table 1 into Eq. (42) and dividing (42) by the square of the current j_{dark}, we obtain the spectral density of its fractional fluctuations in the form

$$S_{\delta j/j}(f)=(j_{dark})^{-2}S_{\delta j}$$
$$=(6.2\cdot10^{-3}\cdot0.2\text{eV}/f\,5.5\cdot10^{5}\text{eV})(0.1\cdot1.6\cdot10^{-19}\text{J}/2\cdot5.7^{2}\cdot10^{-16}\text{m}^{2}\cdot0.9\cdot10^{-30}\text{kg})^{1/2}(1.6\cdot10^{-19}\text{C}/2\cdot10^{-6}\text{A}). \tag{43}$$

Therefore, with the effective mass of the electrons in the InP barrier $m=0.077m_o$, we get a 48 times larger result

$$S_{\delta j/j}(f) =(2.25\cdot10^{-9}/f)(1.66\cdot10^{12})(8\cdot10^{-14})48 =1.44\cdot10^{-8}/f. \tag{44}$$

Amplifying (12) with the square of j_{dark} $=1.25\cdot10^{13}$/s we obtain the spectral density of dark current fluctuations

$$S_{\delta j}(f) =(2.25\cdot10^{-9}/f)(1.66\cdot10^{12})(1.25\cdot10^{13}/s^2)=(2.26\cdot10^{18}/s^2)/f. \tag{45}$$

The corresponding power spectral density $S_{\delta I}(f)$ of electrical current fluctuations δI, and rms noise current I_n are

$$S_{\delta I}(f) =(1.6\cdot10^{-19}\text{C})^2(4.7\cdot10^{16}/s^2)/f = 5.73\cdot10^{-20}\text{A}^2/f \quad \text{and} \quad I_n=2.4\cdot10^{-10}\ \text{A}/f^{1/2}. \tag{46}$$

6.3. Comparison with the experiment

Finally, an experimental value for the current noise (I_{noise}) is obtained by means of the detectivity and responsivity data reported by Jutao Jiang. The NEP=P_{equiv} is the optical power producing a signal equal with the rms noise current. The detectivity (D^*) is proportional with the inverse of the noise equivalent optical power NEP=P_{equiv},

$$D^* = \frac{1}{P_{equiv}} \tag{47}$$

where we have assumed unit area of the detector and a detector electrical bandwidth of 1 Hz. On the other hand, the responsivity (R) is defined as the current generated per optical power unit, and therefore

$$R = \frac{I_{noise}}{P_{equiv}} \tag{48}$$

This implies that the current noise can be written

$$I_{noise} = \frac{R}{D^*} \quad \text{or, equivalently,} \qquad \log I_{noise} = \log R - \log D^* \tag{49}$$

The data gives the following relations for detectivity and responsivity, both as functions of the bias voltage

$$\log D^* = 9 - \frac{V}{2volts} \tag{50}$$

$$\log R_{peak} = -2 + \frac{3V}{4volts} \tag{51}$$

which can easily be applied to equation (45):

$$\log I_{noise} = \log R - \log D^* = \left(-2 + \frac{3V}{4volts}\right) - \left(9 - \frac{V}{2volts}\right)$$
$$= -11 + \frac{5V}{4volts} \tag{52}$$

For V=2 volts this experimental result is within one order of magnitude slightly larger than our theoretical result of $2.4 \cdot 10^{-10}$ A/$f^{1/2}$obtained in (36) above from first principles, without any adjustable parameters. Furthermore, the observed dependence on the voltage in (52) is proportional to $V^{1.25}$ in the rms noise current, i.e., proportional to $V^{2.5}$ in the spectral density. In the Q1/f theory result (42), the last 3 factors are proportional to $V^{1/2}$, V, and V respectively, resulting in a total proportionality to $V^{2.5}$ for V not too small, in nice agreement with the experiment.

Often the collisions will be important. To investigate the collision-dominated case, we will consider the drift velocity of the charge carriers to be $v_d = \mu E$, where μ is the mobility of the charge carrier. The spectral density is given by

$$S_{\frac{\delta\rho}{\rho}} = S_{\frac{\delta J_{dark}}{J_{dark}}} = \frac{4\alpha}{3\pi fN}\left(\frac{eEL_p + W_0}{m^* c^2}\right) = j^{-2} S_{\delta J_{dark}}(f), \tag{53}$$

where total dark current is $J_{dark} = n_o A v_d = n_o A \mu E$. The number of charge carriers N can be written as $n_o AL$ where n_o is the doping level of the well. L is the total length of the device and L_p is the length of the periodicity, or the length of the well and barrier. The effective mass, m^*, is related to the mass of the free particle by $m^* = 0.077 m_0$. The spectral density is then

$$S_{\frac{\delta J_{dark}}{J_{dark}}} = \frac{4\alpha}{3\pi f n_0 AL}\left(\frac{eEL_p + W_0}{m^* c^2}\right). \tag{54}$$

This yields results similar to Eq. (46).

6.4. Conclusions

From (52), (46), and (42) we see that the quantum 1/f formulas correctly describe the 1/f noise in the dark current of QWIPs, both in magnitude and in its dependence on bias. The photo-generation process does not have quantum 1/f noise [17]. These new analytical formulas allow for the first time optimization of the QWIPs with respect to parameters such as choice of the semiconductors used, i.e., the effective mass of the carriers, band and subband parameters, well geometry, etc.

7. Resonant Tunneling Diodes as Thz Generators and their Phase Noise

For imaging and remote sensing applications, the phase noise of the THz generator must be minimized. This can be done as part of the optimization process on the basis of a properly chosen figure of merit. For this, the present paper provides an analytical phase noise expression as a function of the device parameters and operation point. The quantum 1/f theory is used to calculate from first principles the 1/f noise present in the device parameters and in the resulting system frequency from resonant tunneling diodes (RTDs), Gunn devices (TEDs), and transit time diodes. In general, quantum /f fluctuations in the dissipative elements lead to a Q^{-4} dependence of the spectral density of fractional frequency fluctuations and of the corresponding phase noise, where Q is the quality factor.

Resonant tunneling diodes have been proposed as generators of THz oscillations and radiation. They consist of two potential barriers enclosing a quantum well. Electrons penetrating the potential barriers by tunneling are controlled by the quasistationary energy levels defined by the potential well. If their energy is close to the first energy level in the well, resonance occurs, and a peak I_P of the current through the diode occurs. This corresponds to an applied bias voltage V_P. If, however, the applied voltage increases further, only a negligibly small non-resonant current trickle remains at the voltage $V=V_V$ and a broad valley is observed in the I/V characteristic. Scattering processes that reduce the energy of the carriers to a value close to eV_P will always be present, generating a finite current minimum I_V at V_V. Between V_P and V_V there is a negative differential conductance

$$G=-(I_P-I_V)/(V_V-V_P) \qquad (55)$$

on the I/V curve, that is used to generate oscillations. The 1/f noise in I_V is given by Eq. (1) above with

$$(\Delta \mathbf{v}/c)^2=2eV_V/m. \qquad (56)$$

Taking for instance V_P=0.4 V, I_P=2.5 10^8A/m^2, V_V=0.6 V, I_V=4 10^7 A/m^2, we obtain with m_{eff}=0.068 m_o,

$$N I_V^{-2} S_{I_v}(f) = 2\alpha A/f = (7.4/0.068)\ 10^{-9} = 1.3\ 10^{-7}\ /\mathrm{Hz} \tag{57}$$

N is given by

$$N = \tau I_V/e, \tag{58}$$

where τ is the lifetime of the carriers. With a cross-sectional area of 10^{-6} cm^2 and $\tau=10^{-10}$, we obtain N= 2.5 10^6.
Finally, the quantum 1/f frequency fluctuations can be obtained from the formula

$$S_{\delta\omega/\omega} = (1/4Q^4) S_{\delta G/G} \tag{59}$$

which is derived below. This finally yields with Eq. (51)

$$S_{\delta\omega/\omega} = (1/4Q^4)[I_V/(I_P - I_V)]^2 (4\alpha/3\pi)(2eV_V/mc^2) \tag{60}$$

for the fractional frequency fluctuation spectrum exhibited by the RTD, if included in an RF circuit of quality factor Q. For $Q=10^3$ and f=1Hz, we get $S_{\delta\omega/\omega}=5\cdot10^{-28}$/Hz. At $\nu=10^{11}$Hz, the RTD oscillator would have the $1/\omega^3$ phase noise $S_{\Delta\phi}(1\mathrm{Hz}) = 2\cdot10^{-4}$/Hz, or $\mathcal{L}(1\mathrm{Hz})$ =-40dBc/Hz.

8. Frequency and Phase Fluctuations from 1/f Noise in Dissipation

Resonant systems can be described as a harmonic oscillator with losses

$$dx/dt + \gamma dx/dt + \omega_o^2 x = F(t). \tag{61}$$

The quantum 1/f fluctuations are present in the loss coefficient γ. They are given by an expression of the form

$$S_{\delta\gamma/\gamma}(f) = \Lambda/f, \tag{62}$$

where Λ is a quantum 1/f coefficient characterizing the elementary loss process.
The resonance frequency is given by

$$\omega_r^2 = \omega_o^2 + \gamma^2. \tag{63}$$

The quantum 1/f fluctuations in the resonance frequency are given by $\omega_r\delta\omega_r = -2\gamma\delta\gamma$, or

$$\delta\omega_r/\omega_r = -2(\gamma/\omega_r)^2 \delta\gamma/\gamma = -(1/2^2)(\delta\gamma/\gamma), \tag{64}$$

where $Q=\omega_r/2\gamma$ is the quality factor. Squaring, averaging and particularizing Eq. (64) for the unit frequency interval, we obtain the Q^{-4} law

$$S_y(f) \equiv S_{\delta\omega/\omega}(f) \equiv <(\delta\omega_r/\omega_r)^2>_f = (1/4Q^4)<(\delta\gamma/\gamma)^2>_f \equiv \Lambda/(4fQ^4), \quad (65)$$

where y is the fractional frequency fluctuation $\delta\omega_r/\omega_r$. This law is also applicable to quartz resonators, where it was first introduced [5] in 1978. The phase noise is obtained from

$$S_\phi(f) = (\omega_r/2\pi f)^2 S_y(f), \quad \text{or} \quad L(f) = (1/2)S_\phi(f); \quad \mathcal{L}(f)=10 \lg L(f), \quad (66)$$

the middle form being a two-sided power spectrum of phase noise in radians^2/Hz and the last form being expressed in dBc. Here lg designates the decimal logarithm. The examples in Sec. 6 and 7 show how the phase noise is found in other solid state sources of THz radiation.

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