Spontaneous Formation of Horizontal Space Charge Layers in Clouds[']

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Abstract

The origin of the multiple horizontal charge layers observed in the trailing stratiform region of mesoscale convective systems is explained for the first time on the basis of energy considerations and stability calculations. The latter indicate the presence of a fundamental cloud-instability with respect to formation of horizontal polarization layers and associated space charge layers. The author's polarization catastrophe theory of atmospheric electricity has inspired these calculations. This is consistent with the author's earlier derivation of charges in thunderclouds, in growing stratus clouds, in growing and dissipating stratus clouds, or in cosmos.

1. Introduction

For the first time we present rigorous proof of the instability of clouds containing small ice crystals, or sufficiently small H_2O aggregates in general, to the spontaneous formation of several horizontal polarization layers and space charge layers. Our investigation was inspired by the results of balloon observations (schematically shown in Figs. 1 and 2) of many horizontal space charge layers in the trailing region of Mesoscale Convective Systems (MCS).

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The basic structure of MCS convective structures already consists of a stack of four charge regions. Still in the convective region, but outside the updrafts, two more charge regions were found by Rust, et al (1996). This is observed also in the stratiform region.

The method used here was suggested by the author's polarization catastrophe approach to cloud and thundercloud electrification (Handel, 1984, 1985), which later was also applied to the rings of Saturn (Handel and James, 1983), or to planetary atmospheres.

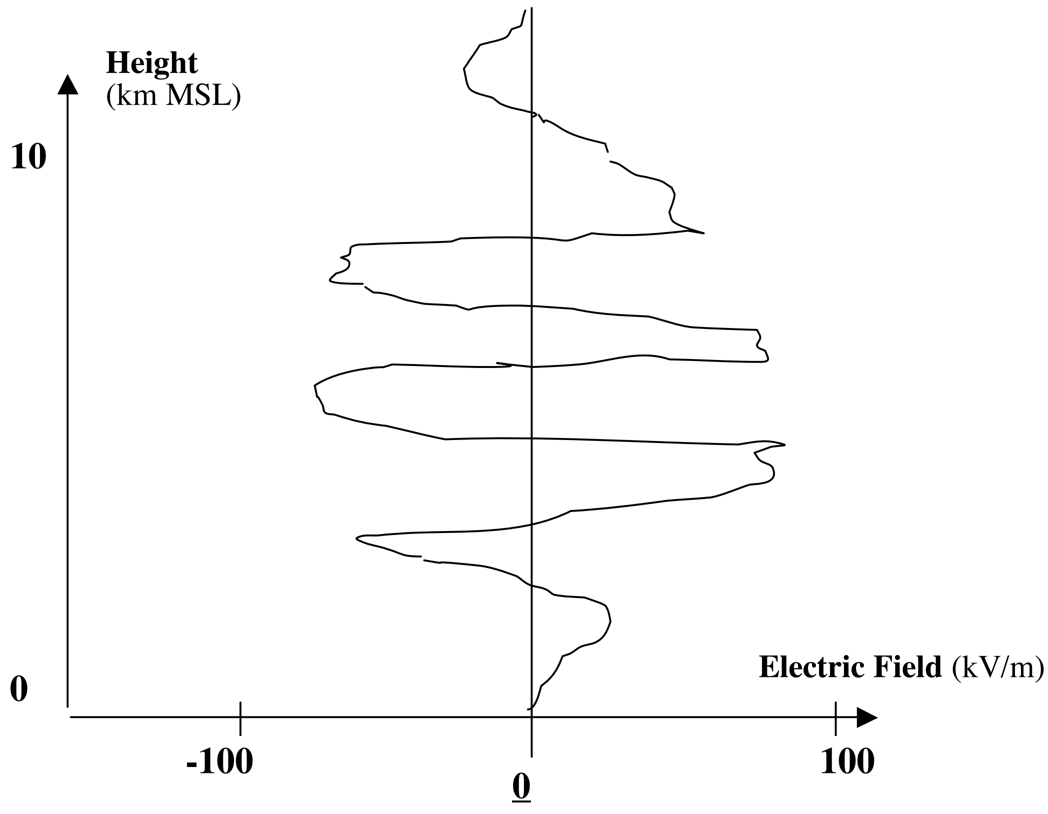

Fig. 1: Horizontal space charge layers found in MCS stratiform regions, representative curve displaying a typical variation of the electric field strength with altitude.

Our proof consists of two parts. In the first part we start from the dipole moment of a single ice crystallite, and derive the expression of the energy of a cloud of such crystallites. We then prove that the energy of a uniform cloud with sufficient ice crystallite concentration is lowered by the spontaneous formation of layers of polarization and polarization charge, the latter being compensated by real masking charges attracted after the updraft phase, from the surrounding air masses.

As shown in Eq. (5) below, the energy is given by the integral from the cloud base to clod top, of $n/\alpha - n^2/3\varepsilon_0$, where α is a polarizability constant of the individual ice crystallite, and $n = n(z)$ is the initially constant concentration of ice crystallites as a function of height z. We denoted with ε_0 the vacuum permittivity. The author's well-known simple polarization catastrophe criterion, requiring a sufficient concentration of crystallites (Eq. 6) is usually

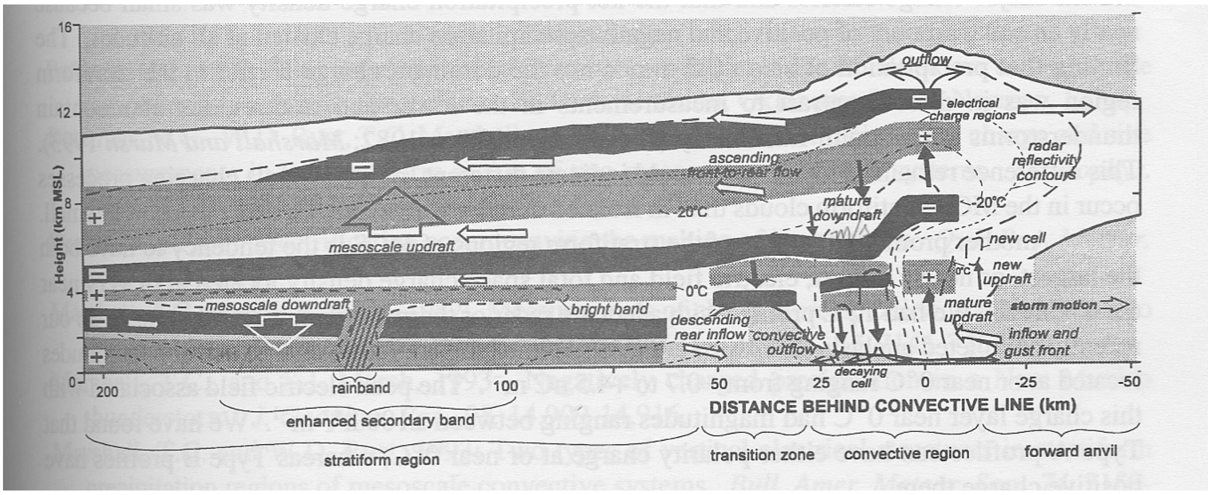

Fig. 2. Electrical model of mesoscale electric systems, indicating the presence of 6 charge layers of alternating sign in the stratiform region (Stolzenburg 1996).

satisfied in most clouds, and this is compatible with the present derivation.

<u>In the second part</u>, we simply show that an arbitrarily small sinusoidal displacement of the cloud results in a perturbation of the concentration of crystallites that lowers the cloud energy, and will grow exponentially. Furthermore, starting more formally from a cloud-lagrangian that incorporates (among other contributions) also the energy contributions derived before, we show that an arbitrarily small sinusoidal perturbation of the cloud concentration will grow exponentially, indicating the presence of the instability that had to be proven.

2. Perturbation lowers the energy of a cloud of ice crystallites

Let n be the concentration of polarizable ice crystallites, each of them in the local field

$$E_{loc} = E_o + P/3\varepsilon_o, \tag{1}$$

where E_o is the macroscopic electric field in the cloud, and P is the cloud polarization, defined as the dipole moment per m^3. Considering each crystallite as comprising two charges +q and -q separated by the dipole length x, we obtain the average contribution of the dipole moment dipole moment of an ice crystallite to the polarization vector

$$p = qx = \alpha E_{loc} = \alpha F/q, \tag{2}$$

where F is the force opposing the separation of the charges in the crystallite. The work u' done in creating the dipoles in the ice crystals per unit cloud volume is

$$u' = n\int F dx = (nq^2/\alpha)\int x dx = nq^2x^2/2\alpha = np^2/2\alpha = P^2/2n\alpha. \tag{3}$$

This increases the energy of the cloud, just like the energy of a set of extended springs. Here P =np. On the other hand, the cloud loses energy due to the orientational polarization of the crystallites that tend to align themselves in a minimum energy direction in the local field

$$u'' = \int E_{loc} dP = \int (E_o + P/3\varepsilon_o) = P^2/6\varepsilon_o + \int E_o dP. \tag{4}$$

The energy of the cloud is thus

$$u = u' - u'' = (p^2/2)[n/\alpha - n^2/3\varepsilon_o]. \qquad (5)$$

The applied macroscopic electric field E_o also contains a polarization induced component, that was assumed to be largely compensated by the masking charges of ions attracted to the regions where the divergence of **P** differs from zero, and will be neglected. A more detailed analysis shows that indeed, for processes that are not too fast, it is correct to assume E_o to be small, while E_{loc} may not be small. Here we may assume that the cloud polarization **P** in Eq. (4) is saturation polarization **P** =n**p**, with p being the total dipole moment of each crystallite. Indeed, the author's polarization catastrophe criterion is (Handel, 1984, 1985; Handel and James, 1983)

$$\nu^2 n > 2.5 \cdot 10^{21}\ \mathrm{cm}^{-3}, \qquad (6)$$

where ν is the number of water molecules in each crystallite and n is their number per unit cloud volume. This criterion is satisfied in most clouds. However, *our derivation is more general, and applicable for any kind of polarization of the individual crystallite*.

Let us apply to all crystallites in the cloud a local displacement field

$$\xi(z, t) = \xi_o(t) \sin kz \qquad (7)$$

with an arbitrarily small amplitude $\xi_o(t)$. The whole cloud was considered to be initially homogeneous, with $n(z)=n_o$ assumed to be initially constant over the whole cloud, from z=0 at the cloud base, to z =h at cloud top. Then, from the equation of continuity, we obtain in first order the concentration perturbation

$$\delta n = -(d/dz)(\xi n_o), \text{ i.e, } n(z,t) = n_o - n_o\xi_o(t)\, k \cos kz. \qquad (8)$$

Substituting into the expression of the relevant energy of the cloud, obtained by integrating Eq. (5),

$$U = (p^2/2)\int[n/\alpha - n^2/3\varepsilon_o] \qquad (9)$$

we obtain

$$\delta U = (p^2/2)\int\left[-(n_o\xi_o/\alpha)\, k \cos kz + (2\xi_o/3\varepsilon_o)\, k \cos kz - (\xi_o^2 k^2 n_o^2/3\varepsilon_o) \cos^2 kz\right]dz \qquad (10)$$

Averaging over z, we obtain, at least for $k=2\pi r/h$ with integer r, $<\cos kz>=0$, and $<\cos^2 kz>=1/2$. Considering first $\xi_o(t)$ constant, we obtain a negative result

$$<\delta U> = -(p\xi_o k n_o/2)^2/3\varepsilon_o \qquad (11)$$

Therefore, the creation of regions of enhanced polarization sandwiching regions of reduced polarization, lowers the cloud energy. This concludes the elementary proof.

We now find the time dependence of $\xi_o(t)$, using the lagrangian approach.

3. Lagrangian approach

The hamiltonian and the lagrangian are given by

$$H = \int \mathcal{H} dz = \int \Pi^2 dz/2nm + \iint \rho_p(z)\rho_p(z')dzdz'/2|z-z'| + (p^2/2)\int[n/\alpha - n^2/3\varepsilon_o]dz; \qquad (12)$$

$$L = \int \mathcal{L}dz = \int nm(d\xi/dt)^2 dz/2 - \iint \rho_p(z)\rho_p(z')dzdz'/2|z-z'| - (p^2/2)\int[n/\alpha - n^2/3\varepsilon_o]dz, \quad (13)$$

where H and L are the corresponding hamiltonian and lagrangian densities, $\Pi = n(z,t)md\xi/dt$ is the momentum density, m the mass of a crystallite, $\rho_p(z)$ is the polarization charge. The latter is again considered to be compensated by the real charge of congregating ions, and is therefore neglected. We therefore neglect the central term. Substituting n from Eqs. (8) and (11) and averaging,, we obtain

$$<\mathcal{L}> = <(m/2)n_o[1-\xi_0(t)\,k\cos kz](d\xi_0/dt)^2\sin^2 kz - (p^2/2)[n/\alpha - n^2/3\varepsilon_o]> - (n_0{}^2p^2/2)[n/\alpha - n^2/3\varepsilon_o]$$

$$-(p\xi_o kn_o/2)^2/3\varepsilon_o. \quad (14)$$

Expressing $\sin^2 kz$ through $\cos^2 kz$, we notice that odd powers of cos kz yield zero in average. The motion is obtained from

$$<(d/dt)[\partial\mathcal{L} / (d\xi_0(t)/dt)] - \partial\mathcal{L} /\partial t> = 0, \quad (15)$$

i.e,

$$d^2\xi_o/dt^2 - (pkn_o)^2\xi_o/3m\varepsilon_o = 0 \quad (16)$$

Thus,

$$\xi_0(t) = \xi_{00}\exp(pn_okt/3^{1/2}m^{1/2}\varepsilon_o{}^{1/2}). \quad (17)$$

This represents an exponentially increasing solution, and thus proves the presence of the instability. The exponential increase with rate $pn_o k/3^{1/2}m^{1/2}\varepsilon_o^{1/2}$ is valid in the linear approximation applicable at the beginning, causing arbitrarily low perturbations to grow. Saturation will be achieved at finite value of the spatially periodic perturbation, leading to a static polarization density wave present spontaneously in the cloud, with a compensating real charge density wave masking it. This can explain the stratification with about six alternating charge layers that has been observed in the stratiform region, in the post-draft part of the convective region, and in their transition region noticed in MCS structures.

However, balloon measurements will yield a measure of the local field given by Eq.(1), present in a spherical cavity inside the cloud, containing the field mill. Furthermore, while the cloud is dissipating (or growing), the compensation of the macroscopic field E_o will never be perfect. Therefore, the measured field in the many balloon soundings will also contain an important component caused by the variation of polarization in time, and by the time lag between that and the adjustment or dissipation of the compensating charges. Both components mentioned here should have the spatial periodicity $2\pi/k$ of the strongest instability, that had the fastest growth rate, taking into account also the diffusional slowing-down, that includes the charge migration process. This is expected to limit the magnitude of the fastest unstable mode k.

4. Conclusions

In most quiescent clouds we expect a tendency to stratification with alternating positive and negative charge layers, due to an inherent instability, first introduced and derived here. The stratification process may be triggered by small initial charge and polarization fluctuations. More work is needed to establish the kinetics, i.e., the development of the electric stratification in time.

It is important to mention that several electrification processes seem to contribute in addition to the polarization effects introduced here and in this author's references given above. Both the latter and the additional processes, mainly non-inductive, may contribute to pinning the polarization layer structure to the temperature scale, notably to the 0^o C isothermal plane.

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