Quantum 1/f Noise and Quantum 1/f Phase Noise Related to the Uncertainty Relations

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Quantum 1/f noise is the manifestation of the coherent and conventional quantum 1/f effects (Q1/fE) [1].

The **conventional Q1/fE** is a fundamental quantum fluctuation of physical cross sections σ and process rates Γ, caused by the bremsstrahlung (recoil) energy and momentum losses of charged particles, when they are scattered, or accelerated in any way. It is given by the spectral density of fractional fluctuations [1]

$$S_{\delta\sigma/\sigma}(f) = (4\alpha/3\pi Nf)(\Delta\mathbf{v}/c)^2, \qquad (1)$$

where $\alpha=e^2/\hbar c=1/137$ is the fine structure constant, $\Delta\mathbf{v}$ is the vector velocity change in the scattering process considered, and N is the number of particles used to define both the notion of current, and the cross section σ.

The closely related **coherent Q1/fE** is present in any current carried by many particles. This effect arises in a beam of electrons (or other charged particles propagating freely in vacuum) from the definition of the physical electron as a bare particle plus a coherent state of the electromagnetic field. It is caused by the energy spread characterizing any coherent state of the electromagnetic field oscillators, according to the **uncertainty principle,** because the phase of the harmonic oscillators is known. This knowledge precludes exact knowledge of the energy, according to **Heisenberg's uncertainty relation**. The energy spread spells non-stationarity, i.e., fluctuations. To find the spectral density of these inescapable basic fluctuations, which are known to characterize any quantum state which is not an energy eigenstate, we use an elementary physical derivation based on Schrödinger's definition of coherent states, followed by a rigorous derivation from a well-known quantum-electrodynamical branch-point propagator.

The coherent quantum 1/f effect will be derived in three steps: first we consider a hypothetical world with just a single mode of the electromagnetic field coupled to a beam of charged particles; considering the mode to be in a coherent state, we calculate the autocorrelation function of the quantum fluctuations in the particle-density (or concentration) which arise from the nonstationarity of the coherent state. Then we calculate the amplitude with which this one mode is represented in the field of an electron, according to Coulomb's law. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the electromagnetic field be characterized by the wave vector **q**, the angular frequency ω = cq and the polarization λ, in short by "q". The coherent state [1] of amplitude $|z_q|$ and phase arg z_q then has the form

$$|z_q\rangle = \exp[-(1/2)|z_q|^2] \exp[z_q a_q^+] |0\rangle$$

$$= \exp[-(1/2)|z_q|^2] \left[\sum_{n=0}^{\infty} (z_q^n)/n!\right]|n\rangle. \qquad (2)$$

Here a_q^+ is the creation operator which adds one energy quantum to the energy of the mode, and all sums go up to ∞. Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n\rangle = (2^n n!\sqrt{\pi})^{-1/2} \exp[-x^2/2]\, H_n(x)\, e^{in\omega t}. \qquad (3)$$

This yields for the coherent state $|z_q\rangle$ the representation $\psi_q(x)=$

$$\exp[-(1/2)|z_q|^2]\exp[-x^2/2]\sum_{n=0}^{\infty} [z_q e^{i\omega t}]^n/[n!(2^n\sqrt{\omega})]^{1/2}H_n(x)$$

$$=\exp[-(1/2)|z_q|^2]\exp[-x^2/2]\exp[-z_q^2e^{-2i\omega t} + 2xz_qe^{i\omega t}]. \qquad (4)$$

In the last form the generating function of the Hermite polynomials was used. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time t or the phase of z_q, is, for $|z_q|<<1$, $P_q(\tau,x) = \langle|\psi_q|_t^2 \mid \psi_q|_{t+\tau}^2\rangle$

$$= \{1 + 8x^2|z_q|^2[1 + \cos\omega\tau] - 2|z_q|^2\}\exp[-x^2/2]. \qquad (5)$$

Integrating over x from -∞ to ∞, we find the autocorrelation function

$$A^1(\tau) = (2)^{-1/2}\{1 + 2|z_q|^2\cos\omega\tau\}. \qquad (6)$$

We now determine the amplitude z_q by Fourier expanding the electric potential e/r of a charged particle in a box of volume V: $|z_q|^2 = \pi(e/q)^2(\hbar cqV)^{-1}$.

Considering now all modes of the electromagnetic field, we obtain from the single - mode result of Eq. (6)

$$A(\tau) = C\,\Pi_q\{1 + 2|z_q|^2\cos\omega_q\tau\} \qquad (7)$$

$$=C\{1+\Sigma_q 2|z_q|^2\cos\omega_q\tau\}=C\{1+4(V/2^3\pi^3)\int d^3q|z_q|^2\cos\omega_q\tau\}$$

Here we have again used the smallness of z_q and we have introduced a constant C. Using the $|z_q|^2$ expression,

$$A(\tau) = C\{1 + 4\pi(V/2^3\pi^3)(4\pi/V)(e^2/\hbar c)\int(dq/q)\cos\omega_q\tau\}$$
$$= C\{1 + 2(\alpha/\pi)\int\cos(\omega\tau)d\omega/\omega\}. \qquad (8)$$

The autocorrelation and spectrum $S_{\delta j/j}(f)$ of the relative density or current $j=e(k/m)|\psi|^2$ fluctuations is the second term in curly brackets divided by the first term

$$S_{|\psi|^2}\langle|\psi|^2\rangle^{-1} = S_j\langle j\rangle^{-2} = 2(\alpha/\pi fN) = 4.6\cdot 10^{-3} f^{-1}N^{-1}. \qquad (9)$$

In conclusion, the paper will show how the uncertainty relations yield both the coherent and conventional Q1/fE.

References

[1] P.H. Handel: "Noise, Low Frequency", *Wiley Encyclopedia of Electrical and Electronics Engineering*, vol. 14, pp. 428-449, John Wiley & Sons, NY, 1999.