Quantum 1/f Effect Based on Quantum Information Theory

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We start here first with a schematic derivation of the entropy of a Quantum 1/f Noise state (see Q1/f [1], this volume) which clarifies the essential elements. Let's simplify our world and assume only one electromagnetic mode of the universe would be present, with frequency ω and wave vector k. Consider the field mode in its ground state and a pair of 2 incoming identical charged particles with the same well-defined wave vector are being scattered by some potential. Both the initial (incoming) and the final (scattered) state represent a pure state. The initial state is $|{+}{+}{-}\rangle_0$, where we reserved the first two arguments of the ket for the two electrons and the last (-) for the field. Due to the interaction with the field, the final state will be

$|f\rangle = (|{+}{+}{-}\rangle + \gamma|{+}{-}{+}\rangle/\sqrt{2} + \gamma|{-}{+}{+}\rangle/\sqrt{2})/(\sqrt{1+\gamma^2})$, (3)

where γ is the emission amplitude of a bremsstrahlung photon, i.e., for the excitation of the field oscillator from its ground state (-) to its first excited state (+). The first two arguments label the state of the charged particles, indicating the presence of an energy loss with (-) and the persistence in the same energy state with (+). The third argument of the kets always labels the field oscillator, as mentioned.

The corresponding density operator of the pure state obtained is $\rho = |f\rangle\langle f|$. Its quantum von Neumann entropy $S/k = \sigma = -\mathrm{Tr}(\rho \log \rho)$ is zero.

Ignoring the field oscillator, i.e., taking the trace of over the field oscillator label, we obtain a classically correlated system, a mixture of 2 pure states, described by the density operator

$[|{+}{+}\rangle\langle{+}{+}| + \gamma^2(|{+}{-}\rangle + |{-}{+}\rangle)(\langle{+}{-}| + \langle{-}{+}|)/2]/(1+\gamma^2)$. (4)

The second term gives the Q1/f noise at $f=\omega/2\pi$, as we show in "Q1/f" in this volume, or below. The corresponding entropy is

$\log(1+\gamma^2) - (\gamma^2\log\gamma^2)/(1+\gamma^2) > 0$ for $0\leq\gamma^2\leq 1$.

The entropy of the system thus appears to have increased although according to quantum mechanics it can not increase. Indeed, according to quantum mechanics, time evolution of a state occurs through a unitary transformation. The latter, however, is known to leave $\sigma = -\mathrm{Tr}(\rho \log \rho)$ invariant.

The QIT [26] allows for a solution of this contradiction, as we prove for the first time here. When two systems A and B with quantum von Neumann entropy $\sigma(A) = -\mathrm{Tr}_A(\rho_A\log\rho_A)$ and $\sigma(B) = -\mathrm{Tr}_B(\rho_B\log\rho_B)$ form a composite system AB of entropy $\sigma(AB) = -\mathrm{Tr}_{AB}(\rho_{AB}\log\rho_{AB})$, we can prove that we have to write

$\sigma(AB) = \sigma(A) + \sigma(B|A) = \sigma(B) + \sigma(A|B)$, (5)

in perfect analogy with classical entropies or information. We introduced $\sigma(A|B)$ as the von Neumann entropy of A conditional on B, or the entropy of A when we know B:

$\sigma(A|B) = -\mathrm{Tr}_{AB}[\rho_{AB}\log\rho_{A|B}]$. (6)

Here we have introduced the conditional density matrix $\rho_{A|B} = \rho_{AB}(1_A \;\; \rho_B)^{-1}$, the quantum analog similar to the classical conditional probability, where   is the tensor product in the joint Hilbert space and $\rho_B=\mathrm{Tr}_A(\rho_{AB})$ is the marginal density matrix obtained by taking a partial trace over the variables of A.

Negative Entropy. The conditional entropy is usually negative in quantum entangled states. Finally, we introduce the quantum mutual entropy which represents the shared entropy, corresponding to the mutual information between A and B:

$\sigma(A{:}B) = -\mathrm{Tr}[\rho_{AB}\log\rho_{A:B}] \equiv \sigma(A) + \sigma(B) - \sigma(AB)$, (7)

where $\rho_{A:B} = \rho_{AB}(\rho_A \;\; \rho_B)^{-1}$. (8)

Taking all logarithms in the base of 2, the entropies will be expressed in bites. Considering our paradoxical case discussed above for simplicity with γ^2=1, we obtain the following entropy diagram of our quantum mechanically entangled triplet of three systems comprising the charged particles A and B, as well as the field oscillator C:

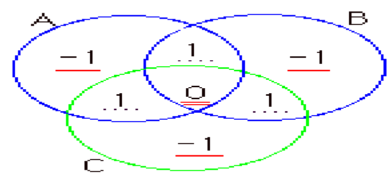

Fig. 1: Entropy diagram of the quantum mechanically entangled triplet of the charged particles A and B, as well as the field oscillator C.

Here the quantum conditional entropies are underlined for emphasis, and the mutual quantum entropy has double underlining. Dotted underlining marks the mutual entropy of two of the subsystems, which is, however, not the mutual entropy of the whole system. We see that A, B and C have each 1 bit of entropy, as we expect, since each can be in a + or - state. In addition, $\sigma_{ABC} = 0$ as expected for a pure state. The negative conditional entropies indicate that this state is not classical.

If we ignore C by tracing over it, we obtain the classically correlated system AB with its positive entropy of 1 *and* the negative quantum entropy state of the ignored field oscillator, conditional on AB:

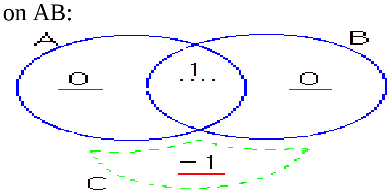

Fig. 2: By tracing over C, we get a classically correlated system AB with its positive entropy of 1 and the negative quantum entropy state of the ignored field oscillator, conditional on AB.

In conclusion, the negative conditional entropy of the ignored field oscillator compensates the positive entropy of the system of two particles. Unitarity and entropy conservation are both satisfied. This is the solution of the paradox. It explains why our earlier explanation of the Q1/fE in terms of a two-particle wave function was correct, in spite of the apparent increase in entropy. The quantum 1/f effect appears to generate entropy, but this is only because we do not observe the negative conditional entropy contribution of the photons, or of the oscillator states present in the photon cloud of a physical charged particle. The latter applies to the coherent Q1/fE [1].

[1] A.G. Tournier and P.H. Handel in this volume.