1/f FREQUENCY FLUCTUATIONS AND PHASE NOISE IN MEMS RESONATORS

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The classical forms of noise present in the frequency of MEMS resonators have been recently investigated and described in detail [1]. The present paper extends this investigation to 1/f noise. That is of fundamental nature in microelectromechanical systems (MEMS) resonators, and is given by the universal quantum 1/f effect [2]. In recent years new piezoelectric and compound semiconductor materials have been developed along with advanced manufacturing technologies for MEMS including them. These technologies enable the deposition of thin compound semiconductor, or PZT films and their integration into MEMS devices [3].

When environmental fluctuations are reduced, e.g., by temperature stabilization, elimination of molecular adsorption, etc., the fundamental limitation introduced by the quantum 1/f effect becomes visible as a flicker floor in the time-domain noise representation. Being ultimately based on Heisenberg's uncertainty relations, this effect defines the quantum limit achievable at low frequencies, when the classical sources of noise have been sufficiently reduced.

The quantum 1/f effect manifests itself in MEMS resonators through conventional quantum 1/f noise. This is a quantum fluctuation present in the quantum mechanical notion of "physical cross section" σ and "physical process rate" Γ, caused by the reaction of the electromagnetic field on the charged particle that emitted it, or in general, on the system that emitted it. It is given by the spectral density of fractional fluctuations

$$S_{\delta\sigma/\sigma}(f) = S_{\delta\Gamma/\Gamma}(f) = (4\alpha/3\pi f)(\Delta\mathbf{v}/c)^2, \qquad (1)$$

where $\Delta\mathbf{v}$ is the velocity vector change of the current carriers of charge e, and $\alpha=e/\hbar c=1/137$ is Sommerfeld's fine structure constant. The rate Γ of interest for the calculation of 1/f resonance frequency fluctuations in A_{III}-B_V or Si MEMS is the rate of phonon removal from the main mechanical oscillation mode of the oscillating bar. Our mechanism of quantum 1/f noise generation is based on bremsstrahlung from electric or magnetic dipoles, rather than from scattering of individual carriers.

The MEMS resonators are shaped as a bar of micro- or nanometric dimensions incased (fixed) at both ends, subject to bending strain. They are driven by an AC current I flowing in a very thin straight wire attached along the bar, in the presence of a strong constant magnetic field **B**, perpendicular to it. Alternatively, they could be driven electrostatically, or capacitively, by using the wire, or a metallic thin sheet deposited along that side of the bar, as one of the two "plates" of a capacitor. An AC voltage would then be applied to the capacitor at the resonance frequency.

The approach developed for the case of resonators based on quartz and other piezoelectric substances can not be applied to Si MEMS resonators, because silicon is not piezoelectric. It could only be applied to piezoelectric MEMS resonators, using, e.g., quartz. We therefore develop a new approach in this paper, applicable to any magnetically driven MEMS resonators. We consider the case of magnetic excitation. The electrostatic case is similar, but noisier. Therefore, we recommend the magnetically driven approach also for piezoelectric MEMS made of compound semiconductors.

The elementary act of dissipation is the absorption of one phonon from the main resonator mode. This can happen either through three photon processes, dominant at higher temperatures, or through two processes, dominant at low temperatures, in the presence of crystal defects of various types. In both cases this elementary act has a spontaneous "bremsstrahlung photon emission (probability) amplitude" associated with it, because it causes a sudden reduction $\Delta\dot{\mathbf{m}}$ of the rate $d\mathbf{m}/dt \equiv \dot{\mathbf{m}}$ of change of the magnetic dipole moment of the system. The system is defined here to include the current carrying circuit, the applied magnetic field **B** with the agency generating it, plus the oscillating bar, i.e., the MEMS. The rate $\dot{\mathbf{m}}$ of change of the magnetic dipole moment of the system is caused by bending oscillations of the bar with the current-carrying wire attached to its side, cutting the lines of force of **B** while it moves.

The conventional quantum 1/f noise in the rate of dissipation Γ is obtained in the CGS Gaussian system by simply replacing the current change $e\Delta\mathbf{v}$ with $\Delta\dot{\mathbf{m}}$

$$S_{\delta\Gamma/\Gamma}(f) = (4\alpha/3\pi f)(\Delta\dot{\mathbf{m}}/ec)^2. \qquad (2)$$

To calculate the change $\Delta\dot{\mathbf{m}}$, we note that $(\dot{\mathbf{m}})^2$ is proportional to the transversal vibration speed of the bar and to the total mechanical energy $W=(n+1/2)\hbar\omega$ in the main oscillation mode of the resonator,

$$(n+1/2)\hbar\omega = \kappa(\dot{\mathbf{m}})^2/2. \qquad (3)$$

Here κ is a coefficient of proportionality easy to calculate. When a phonon is removed from the main MEMS resonator mode, the energy in Eq. (3) is reduced by $\hbar\omega = \kappa\dot{\mathbf{m}}\,\Delta\dot{\mathbf{m}}$.

This yields $(\Delta\dot{\mathbf{m}})^2 = (\hbar\omega/\kappa\dot{\mathbf{m}})^2 = \hbar\omega/(2n+1)\kappa$. Eq. (2) becomes now

$$S_{\delta\Gamma/\Gamma}(f) = (4\alpha/3\pi f)\hbar\omega/(2n+1)\kappa(ec)^2 = \underline{4\omega/3\pi f(2n+1)\kappa c^3} = 2\hbar\omega^2/3\pi f W\kappa c^3. \qquad (4)$$

This quantum 1/f noise, proportional to the resonance frequency and inversely proportional to the number of quanta in the main oscillation mode, decreases when the energy in the main resonator mode increases. This is characteristic of quantum mechanical spontaneous emission noise in general. With $\omega=10^{10}$rad/sec, W=0.01erg, we get $10^{-27}/f\kappa$.

To estimate the coefficient κ, we write the mechanical energy, including both the equally contributing kinetic and elastic components, in the form

$$W = M_{eff}v^2/2, \qquad (5)$$

where M_{eff} is a mass close to the mass M of the bar. Noting that the rate of change in the magnetic moment of the system, caused by the motion of the bar of length L with speed v, is

$$\dot{\mathbf{m}} = \mathbf{n}SdI/dt + nIdS/dt = \mathbf{n}SBLv/cL = \mathbf{n}SBLvc/2\int \ln(R/a_s)ds, \qquad (6)$$

we find

$$\kappa = M_{eff}v^2/\dot{\mathbf{m}}^2 = M_{eff}[cL/SBL]^2. \qquad (7)$$

Here we introduced the area S and the inductance L of the AC circuit, as well as the approximate expression of L in terms of an integral around the whole circuit. This involves the radius a_s of the cross section of the electric wire used for the AC circuit, as a function of the length parameter s along the wire. We neglected the term with IdS/dt, because $S\omega >> Lv$. We can approximate $S/2\int\ln(R/a_s)ds$ with $R/\langle\ln(R/a_s)\rangle$, where $R=S/l$ is the circuit area S divided by its length l. This way we obtain e.g., $R'=S/2\int\ln(R/a_s)ds = 300$ cm as an order of magnitude, and $c/R'=10^8$Hz. With these notations and approximations Eq. (7) becomes $\kappa = M_{eff}[c/R'BL]^2$. With $M_{eff}=10^{-11}$g, $L=10^{-3}$cm and $B=10^4$Gauss, we obtain $\kappa=10^3\text{cm}^{-1}$. Substituting into Eq. (4), we obtain

$$S_{\delta\Gamma/\Gamma}(f) = (4\alpha/3\pi f)(R'BL)^2\hbar\omega/(2n+1)M_{eff}e^2c^4 = \underline{4(R'BL)^2\omega/3\pi f(2n+1)M_{eff}c^5}$$

$$= 2(R'BL)^2\hbar\omega^2/3\pi f W M_{eff}c^5. \qquad (8)$$

For instance, with the above numerical example, with $\omega=10^{11}\text{s}^{-1}$, and with $W=10^{-8}$ erg, we obtain $10^{-22}/f$.

Finally, the quantum 1/f frequency fluctuations can be obtained from the formula

$$S_{\delta\omega/\omega} = (1/4Q^4)S_{\delta\Gamma/\Gamma} \qquad (9)$$

which is derived in Sec. IV below. This finally yields with Eq. (8)

$$S_{\delta\omega/\omega} = (1/2Q^4)(R'BL)^2\hbar\omega^2/3\pi f W M_{eff}c^5 \qquad (10)$$

for the fractional frequency fluctuation spectrum exhibited by the magnetically driven MEMS resonator. With Q=10 and with the above estimate for $S_{\delta\Gamma/\Gamma}(f)$ we obtain $S_{\delta\omega/\omega}=2.5\,10^{-27}/f$. This implies reasonable frequency stability $\delta\omega/\omega$ of about 10^{-13}. The numerical estimate shows that the magnetically driven resonator is much less noisy than the electrostatically driven ones, when it comes to 1/f noise. This is because the magnetic dipole radiation is much weaker, of higher order. PHH acknowledges ONR-MURI 17/2001 support (Dr. H. Dietrich).

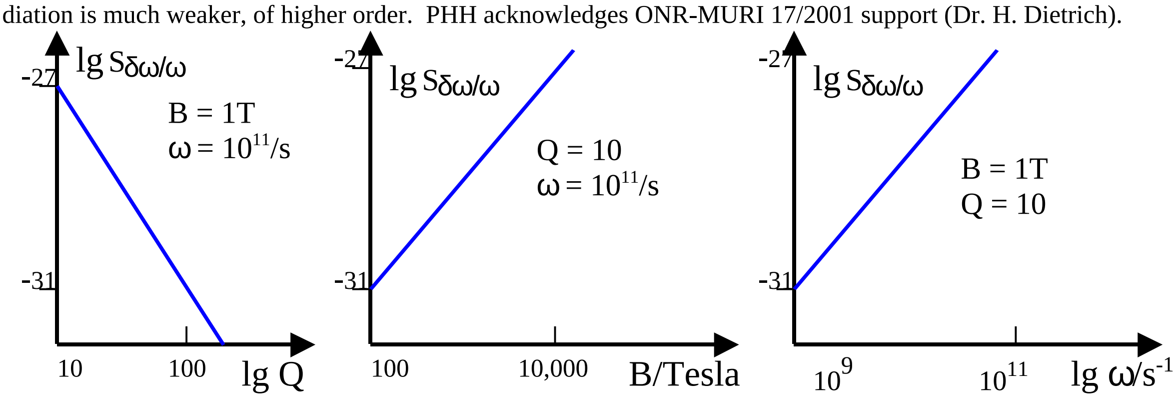

Fig. 1: Dependence of $S_{\delta\omega/\omega}$ on quality factor Q, applied field B, and frequency ω.

References

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