GaN HEMT GEOMETRY AND PIEZOELECTRIC QUANTUM 1/f NOISE

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In general, piezoelectric quantum 1/f noise arises from the reaction of emitted or virtual phonons on the electrons which have emitted them into the crystal lattice. Here we consider the GaN/AlGaN geometry to derive the relation between the coherent and conventional quantum 1/f effects for this special case. We compare it to the practically important similar relation applicable for the usual electromagnetic quantum 1/f effects in these devices, particularly in nano-devices. **Keywords:** GaN HEMT, 1/f Noise.

1. Introduction

The best known form of quantum 1/f noise [1]-[2] is certainly the electromagnetic, or QED form, that has been used to improve semiconductor devices other than the MOSFET and many other high-tech devices, sensors, and systems since the 1980s. Both the coherent and conventional QED-type Quantum 1/f Effect (Q1/fE) contribute to quantum 1/f noise, and are caused by the infrared divergent coupling of charged particles and soft photons. Total 1/f noise is $S_j(f)/j^2 = \alpha_o/Nf$ with α_o defined below as a weighted sum of α_{conv} and α_{coh}.

The conventional Q1/fE is a new property of the "physical cross sections and process rates" introduced by one of us in order to allow quantum mechanics to explain fundamental 1/f noise in materials, devices and systems, including phase noise. It does not depend on the presence of other similar charged particles drifting in the electrical field. For the usual QED case it is given by $\alpha_{conv} = 2\alpha(\Delta\mathbf{v}/c)^2/3\pi$, where α is 1/137.

The coherent Q1/fE, on the other hand, is a basic property of any particle. It is caused by the uncertainty of its rest mass, due to the same infrared divergent coupling and due to the coherent state in which the photon states happen to be. This coherent state of the field is simply due to the presence of the particles and due to their motion. For the usual QED case it is given by $\alpha_{coh} = 2\alpha/\pi$, where the scatter-velocity-change $\Delta\mathbf{v}$ does not enter.

The drift motion of the current carriers, mentioned in the previous paragraph, causes a magnetic field and a magnetic form of kinetic energy, $LI^2/2$ where L is the inductance. In a thick copper wire, it is much larger than the mechanical form of the kinetic energy of the same drift motion of the current carriers. This magnetic energy of the drift motion of a single electron is proportional to the number N' of other electrons, per unit length of the wire, that are drifting along with drift velocity v that is their average velocity. Thus, $LI^2/2$ goes like N'^2. On the other extreme of a nanoscopically thin semiconductor, the mechanical form $mv^2/2$ dominates. That goes like N', without any coherence. Therefore we understand why the coherent Q1/fE is dominant in a copper wire, while the conventional Q1/fE is found to be dominant in the very thin, poorly doped, semiconductor whisker.

The application to QLD allows us for the first time to also explain the giant quantum 1/f noise in large ferroelectric semiconductor samples of BaSrTiO3, GaN and AlGaN. The speed of sound replaces the speed of light in the denominator of α, and the observed 1/f noise is 10^6 times larger, as expected from the theory. We also show that a system of subharmonics generates a similar hierarchy of states as the harmonics can generate.

2. Coherent and conventional Piezoelectric Quantum 1/f Effect

The conventional, or incoherent, piezoelectric quantum 1/*f* noise effect [3], [4] is given for small samples or devices by

$$\langle\sigma\rangle^2 S_\sigma(f) = 2gA/fN \tag{1}$$

with $A = 2(\Delta\mathbf{v})^2/3\pi(v_s)^2$, $\Delta\mathbf{v}$ is the change of electron velocity during scattering and σ is any scattering cross section defined for carriers that exhibit piezoelectric coupling to ELF phonons. Here g is the dimensionless piezopolaron coupling constant, similar to the fine structure constant α, but with the speed v_s of transversal sound waves replacing the speed of light in the denominator. It also involves a geometrical constant of the order of unity, dependent on the lattice. In a first approximation, σ can also be the mobility μ, the conductivity, or the current *j*. In the coherent case [4] of large devices,

$$\frac{S_j(f)}{j^2} = \frac{4g}{\pi f N} \frac{1}{\left[1-\left(\hbar k_s / m^* v_s\right)^2\right]} \equiv \frac{\alpha_{coh}}{fN} \tag{2}$$

Here $|\mathbf{v}| = u$ is the drift velocity of the carriers, m* their effective mass, and v_s is the speed of sound for transversal acoustic phonons. In this section we derive the connection between the coherent and conventional forms of Lattice-Dynamical (piezoelectric) Quantum 1/f Effect (LD-Q1/fE) similarly to the way this was done in 1985 for the usual QED-Q1/fE [3].

In QLD, the amplitude of the piezo-elastic transversal lattice mode with wave vector q is the Fourier component q of the displacement caused by an electron in the lattice, expanded over a cubic box of volume V

$$A_{\vec{q}} = \frac{-i v_s (2\pi g / q^3 V)^{1/2}}{v_s - \hbar \hat{q}\,\vec{k}/m} \tag{3}$$

The part corresponding to the motion is obtained by subtracting the static part (Eq. 3 for k=0) that applies for an electron at rest

$$A_{\vec{q}}^{mot} = -i v_s \left(\frac{2\pi g}{q^3 V}\right)^{\frac{1}{2}} \left[\frac{1}{v_s - \hbar\hat{q}\,\vec{k}/m} - \frac{1}{v_s}\right] = -i v_s \left(\frac{2\pi g}{q^3 V}\right)^{\frac{1}{2}} \frac{\hbar\hat{q}\,\vec{k}}{m v_s (v_s - \hbar\hat{q}\,\vec{k}/m)} \tag{4}$$

The corresponding motional energy W^{mot} for an electron of wave vector k is thus

$$v_s^{-2} W_{\vec{k}}^{mot} = \sum_{\vec{q}} |A_{\vec{q}}^{mot}|^2 \hbar\omega_q = \frac{2\pi g}{V} \frac{\hbar^3}{m^2 v_s} \frac{V}{(2\pi)^3} \int d\Omega \int_0^{q_D} dq \frac{k^2 \cos^2\theta}{(v_s - \hbar k \cos\theta / m)^2} \tag{5}$$

Here q_D is the Debye wave vector. The integration yields

$$W_{\vec{k}}^{mot} = \frac{g\hbar^3}{2\pi m^2 v_s} \frac{V}{(2\pi)^3} \int_0^{q_D} dq \int_{-1}^{1} \frac{k^2 x^2 dx}{(1-\hbar k x / m v_s)^2} = \frac{g}{2\pi} \frac{m v_s^2 q_D}{k} F\left(\frac{v}{v_s}\right) \tag{6}$$

With $q_D = 1/r'_o$, we obtain thus for a 2DEG thin and w cm wide channel, with large scale cylindrical symmetry

$$W_{\vec{k}}^{mot} = \frac{g}{2\pi} \frac{m v_s^2}{k r'_o} F\left(\frac{v}{v_s}\right) \xrightarrow{\text{Cylindrical Symm}} \frac{g}{2\pi} \frac{m v_s^2}{k r'_o} F\left(\frac{v}{v_s}\right) N' [\pi + \ln\frac{R}{r'_o}] \tag{7}$$

N' is again the number of carriers per unit length of the sample in the direction of current flow, and *m** is the electron effective mass. We approximated the inductance integral for the HEMT's 2DEG channel with a simple rectangular field-lines-geometry up to a distance s=w from the channel, with subsequent decrease of the field at R>s>w, as if the large scale symmetry would be cylindrical. Here we introduced the function

$$F(u/ v_s) = 2(u/ v_s)\{ 1 + 1/[1 - (u/ v_s)^2]\} + \ln[1-(u/ v_s)]^2 - \ln[1+(u/ v_s)]^2. \tag{8}$$

We introduce the ratio of the coherent piezoelectric energy of the drift motion divided by the additional kinetic energy of the carriers introduced by the drift velocity u.

$$s' = \frac{\frac{g}{2\pi} \frac{m v_s^2}{k r'_o} F\left(\frac{u}{v_s}\right) N' [\pi + \ln\frac{R}{r'_0}]}{N' m u^2 / 2} = \frac{g}{\pi} \frac{v_s^2}{u^2 k r'_o} F\left(\frac{u}{v_s}\right) [\pi + \ln\frac{R}{r'_0}] \tag{9}$$

This is the effective ratio s' of terms present in the hamiltonian, corresponding in QLD to the parameter s.

For $u/v_s << 1$, $F(u/v_s) = (2/3)(u/v_s)^3$; then *s'* can be simplified as $s' = 2gN'\hbar/3\pi m^* v_s$. This expression can be used for small drift velocity, much smaller than the sound velocity in the semiconductor. For $1-(u/v_s) << 1$, $F(u/v_s) = 1/[1-(u/v_s)]$, then *s'* can be approximated as $s' = (gN'\_/\pi m^* v_s)(v_s/u)^3\{1/[1-(u/v_s)]\}$, which is very large. This expression is used for drift velocities approaching the speed of sound.

As for QED, the coherent and conventional forms of QLD-Q1/fE are superposed with weights s/(1+s) and 1/(1+s). In general, including also intermediary sizes, we have $S_j(f)/j^2 = \alpha/Nf$ with N total carriers, and α given by

$$\alpha = \alpha_{conv} \frac{1}{(1+s')} + \alpha_{coh} \frac{s'}{(1+s')} \tag{10}$$

Again, similar to s, the weight s' is the ratio between the piezoelectric or LD form of kinetic energy [associated with the electric current present in the sample or device, that is coherent], and the incoherent kinetic energy $m_{eff} u^2/2$ of individual particles, [caused by the drift velocity u].

The piezoelectric coupling constant g is dependent on the strain tensor S_{kl} and on the applied field **E**. It can be expressed in terms of the elastic constants c_{ijkl}, the piezoelectric tensor components e_{ijk}, and the dielectric tensor components ε_{ij}, all defined by the piezoelectric equations that give the stress tensor T_{ij} and the electric displacement vector **D**

$$T_{ij} = c_{ijkl}S_{kl} - e_{ijk}E_k \qquad (11)$$
$$D_i = 4\pi e_{ijk}S_{jk} + \varepsilon_{ij}E_j \qquad (12)$$

The dependence of the piezoelectric coupling constant on the shear strain is given by Eq. (13) and graphed in Fig. (1). We notice that the coupling constant increases in the presence of stress. For the n-GaN sample studied by Sia Eng Kee, we see from Fig. 1 that g=8, and therefore the coherent quantum 1/f coefficient is 5. For ELO GaN one obtains g=3 and a quantum 1/f coefficient of α=2. This is in good agreement with the experiment [5].

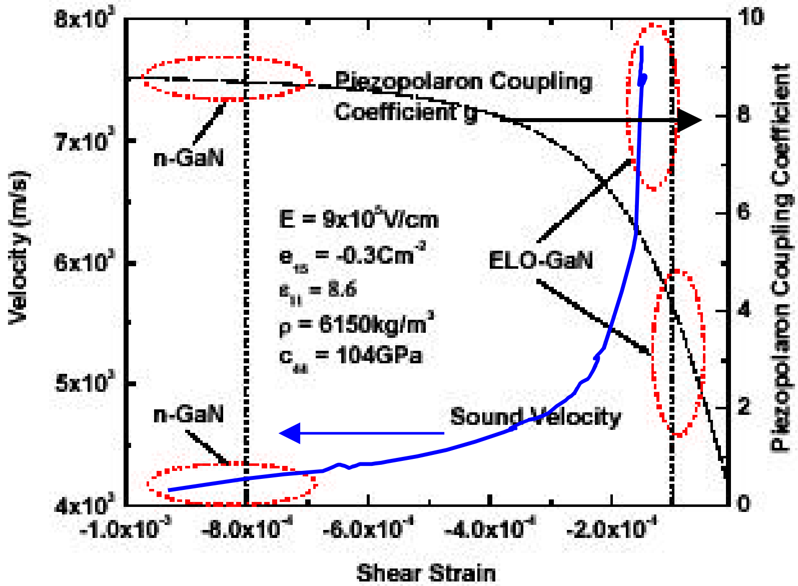

Fig. 1: Strain dependence of the transversal sound velocity and of the piezoelectric coupling constant.

$$g = \frac{e^2}{\hbar\varepsilon_0\left[\left(c_{44} + \dfrac{\varepsilon_0\varepsilon_{11}E^2}{(4\pi S_{11})^2}\right)/\rho\right]^{\frac{1}{2}}}\frac{e_{15}^2}{\varepsilon_0\varepsilon_{11}c_{44}} \qquad (13)$$

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