CONNECTION OF COHERENT AND CONVENTIONAL PIEZOELECTRIC QUANTUM 1/F NOISE AND THE ROLE OF SUBHARMONICS

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Piezoelectric quantum 1/f noise arises from the reaction of emitted or virtual phonons on the electrons which have emitted them into the crystal lattice. Here we derive the relation between the coherent part and the conventional quantum 1/f effects. We find it to be similar to the relation derived earlier for the usual electromagnetic quantum 1/f noise. This is applicable to ferroelectric semiconductors such as barium strontium titanate, GaN, AlGaN. The role of subharmonics is briefly discussed for the lowest frequencies.

1. Introduction

The best known form of quantum 1/f noise [1]-[2] is certainly the electromagnetic, or QED form, that has been used to improve semiconductor devices other than the MOSFET and many other high-tech devices, sensors, and systems since the 1980s. Both the coherent and conventional QED-type Quantum 1/f Effect (Q1/fE) contribute to quantum 1/f noise, and are caused by the infrared divergent coupling between charged particles and soft photons. In final analysis, this new aspect of quantum mechanics is therefore due to the non-linearity of the system of particle and field, where the field reacts on the particles that generated the field in the first place.

The conventional Q1/fE is a new property of the "physical cross sections and process rates" introduced by one of us in order to allow quantum mechanics to explain fundamental 1/f noise in materials, devices and systems, including phase noise. It does not depend on the presence of other similar charged particles drifting in the electrical field.

The coherent Q1/fE, on the other hand, is a basic property of any particle. It is caused by the uncertainty of its rest mass, due to the same infrared divergent coupling and due to the coherent state in which the photon states happen to be. This coherent state of the field is simply due to the presence of the particles and due to their motion.

The energy of coherent states is uncertain, as is known since Schrödinger introduced them in the 1920s. This uncertainty, derived from Heisenberg's uncertainty principle, causes the state of any particle to be non-stationary and beset by the fundamental coherent quantum 1/f noise $2\alpha/\pi f$, where $\alpha=e^2/\hbar c=1/137$ is Sommerfeld's fine structure constant. It is remarkable that the power spectral density of fractional fluctuations $2\alpha/\pi f$ in

current, votage, mobility, diffusion constant, recombination speed, quartz resonator dissipation rate and frequency, etc., etc, contains Planck's constant in the denominator.
That makes this new aspect of quantum mechanics, or most important and fundamental form of quantum chaos, particularly interesting. It is the one that shapes our very existence, and causes us to like spontaneously, for instance, 1/f, or symphonic, music. Epistemologically, quantum 1/f noise is just a special case of the fundamental universal 1/f spectrum, caused by the simultaneous presence of nonlinearity and a special form of homogeneity in any system, e.g., traffic on the highways. This is described by Handel's "Sufficient Criterion," that is satisfied also by QED at low frequencies.

The drift motion of the current carriers, mentioned in the previous paragraph, causes a magnetic field and a magnetic form of kinetic energy, $LI^2/2$ where L is the inductance. In a thick copper wire, it is much larger than the mechanical form of the kinetic energy of the same drift motion of the current carriers. This magnetic energy of the drift motion of an electron is proportional to the number of other electrons, per unit length of the wire, that are drifting along with drift velocity v that is their average velocity. On the other extreme of a nanoscopically thin semiconductor, the mechanical form $mv^2/2$ dominates, as was pointed out already at the start of the theory of fundamental 1/f noise. Therefore we understand why the coherent Q1/fE is found to be dominant in a copper wire, while the conventional Q1/fE is found to be dominant in the very thin, poorly doped semiconductor whisker.

In Sec. 2 below we review the elementary relation between coherent and conventional Q1/fE [3] that is suggested by the relation between the magnetic and kinetic drift energies for the electromagnetic or quantum electrodynamical (QED) case, in which photons are the infraparticles. In Sec. 2 we repeat this more formally. In Sec. 3 we extend this treatment to the case of piezoelectric quantum 1/f noise, in which soft (transversal) piezophonons are the infraparticles. This was also introduced as Quantum Lattice-Dynamical (QLD) quantum 1/f noise. There is one type of quantum 1/f noise for each system of infraquanta with infrared divergent coupling to the current carriers, as Handel pointed out already in his review article at the First International Conference on 1/f Noise in Tokyo, in 1977. Each of these types of quantum 1/f noise, comprise a coherent Q1/fE and a conventional Q1/fE.

The application to QLD allows us for the first time to explain the giant quantum 1/f noise in large ferroelectric semiconductor samples of BaSrTiO3, GaN and AlGaN. The speed of sound replaces the speed of light in the denominator of α, and the observed 1/f noise is 10^6 times larger, as expected from the theory. But there are no low frequency piezophonons, even in the largest semiconductor samples. In Sec. 4 we show that a system of subharmonics generates a similar hierarchy of states as the harmonics can generate. This is intimately related to the realization that nonlinearity causes all fundamental 1/f noise, of any nature.

2. Elementary Derivation of the Parameter s'

We suggest that the relation of the coherent Q1/fE to the conventional Q1/fE is described in a first, rough, approximation through the parameter s defined here. The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current. It is, therefore, the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the hamiltonian by the magnetic energy E_m, per unit length, of the current carried by the sample. In very small samples or electronic devices, this coherent magnetic energy

$$E_m = \int (B^2/8\pi) d^3x = [nevS/c]^2 \ln(R/r) \qquad (1)$$

is much smaller than the total kinetic energy E_k of the drift motion of the individual carriers

$$E_k = \sum mv^2/2 = nSmv^2/2 = E_m/s. \qquad (2)$$

Here we have introduced the magnetic field B, the carrier concentration n, the cross sectional area S and radius r of the cylindrical sample (e.g., a current carrying wire), the radius R of the electric circuit, and the "coherence ratio"

$$s = E_m/E_k = 2ne^2S/mc^2 \ln (R/r) \approx 2e^2N'/mc^2, = 2N'r_o. \qquad (3)$$

Here N' = nS is the number of carriers per unit length of the sample, $r_o = e^2/mc^2 = 2.8 \cdot 10^{-13}$ cm is known as the classical radius of the electron, and the natural logarithm ln(R/r) has been approximated by one in the last form. We expect the total observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN] = [2\alpha/(1+s)fN][A+s/\pi], \qquad (4)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side. In this equation $\alpha A = 2\alpha(\Delta \mathbf{v}/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum 1/f effect. This equation is limited to quantum 1/f mobility (or diffusion) fluctuations, and does not include the quantum 1/f noise in the surface and bulk recombination cross sections in the surface and bulk trapping centers, in tunneling and injection processes, in emission or in transitions between two solids.

Note that the coherence ratio s introduced here equals the unity for the critical value $N' = N'' = 2 \cdot 10^{12}$/cm., e.g. for a cross section $S = 2 \cdot 10^{-4}$ cm^2 of the sample when $n = 10^{16}$. For small samples with N'<<N" only the first term survives, while for N'>N" the second term in Eq. (4) is dominant. In practice, s is therefore twice the number of carriers in a salami slice perpendicular to the direction of the current, with a thickness given by the classical radius of the electron.

3. QED Derivation of the parameter s

In QED, the displacement amplitude of the electromagnetic oscillator with wave vector q and polarization ε is the Fourier component of the vector potential expanded over a cubic box of volume V

$$Z_q = \frac{ek^\mu \varepsilon_\mu}{\vec{k}\,\vec{q} - m\omega_q/\hbar} \frac{1}{\sqrt{2\hbar cqV}} \approx \frac{ek^\mu \varepsilon_\mu}{-mc} \frac{\hbar}{\sqrt{2\hbar cq^3 V}} \qquad (5)$$

It is produced by the presence of an electron of wave vector k, charge e and mass m. It is well known that the electron moving with velocity **v** generates a vector potential

$$A = \frac{e\vec{v}}{4\pi cr} = \frac{\vec{v}}{c}\Phi \xrightarrow{Fourier} \frac{e\vec{v}}{cq^2} = \frac{e\hbar\vec{k}}{mcq^2} \qquad (6)$$

On the right we have shown the Fourier transform, and Φ is the electric potential. This coincides with Eq. (5), if we apply the normalization for Fourier expansion in a box of volume V. The current density of the electron is

$$\vec{j} = e\vec{v}\,\delta(\vec{r}) \xrightarrow{Fourier} e\vec{v} \qquad (7)$$

The motional part of the energy is

$$W_{motion} = \tfrac{1}{2}\int \frac{\vec{j}}{c}\,\vec{A}\,d^3r = \tfrac{1}{2}\frac{\vec{j}}{c} * \vec{A} = \tfrac{1}{2}\int e\vec{v}\,\frac{ev}{cq^2}\,4\pi\,\frac{dq}{(2\pi)^3} = \frac{e^2v^2}{4\pi^2c^2}\,q_{\max} = \frac{e^2v^2}{4\pi^2c^2}\,\frac{\pi}{2r_0} \qquad (8)$$

Here * indicates a convolution and r_o is the classical radius of the electron, introduced in the previous section. The upper cutoff q_{max} introduced here for q, corresponds to a lower cutoff r_o for the first integral in Eq. (8). By carrying out that first integral we get

$$W_{motion} = \frac{e^2v^2}{8\pi r_0 c^2} \qquad (9)$$

and by comparing with the q-space integral, we obtain

$$q_{max} = \pi/2r_o. \qquad (10)$$

We now have all the ingredients for a discussion of the piezoelectric case.

4. Connection of coherent and conventional QLD-Q1/fE

The conventional, or incoherent, piezoelectric quantum 1/*f* noise effect [3], [4] is given for small samples or devices by

$$\frac{S_\sigma(f)}{\langle\sigma\rangle^2} = \frac{Ag}{Nf} \qquad (11)$$

with $A = 2(\Delta v)^2/3\pi(v_s)^2$, Δv is the change of electron velocity during scattering and σ is any scattering cross section defined for carriers that exhibit piezoelectric coupling to ELF phonons. Here g is the dimensionless piezopolaron coupling constant, similar to the fine structure constant α, but with the speed of transversal sound waves replacing the speed of light in the denominator. It also involves a geometrical constant of the order of unity, dependent on the lattice. In a first approximation, σ can also be the mobility μ, the conductivity, or the current *j*. In the coherent case [4] of large devices,

$$\frac{S_j(f)}{j^2} = \frac{4g}{\pi f}\,\frac{1}{\left[1-\left(\hbar k_s / m^* v_s\right)^2\right]} \qquad (12)$$

Here $|\mathbf{v}| = u$ is the drift velocity of the carriers, m* their effective mass, and v_s is the speed of sound for transversal acoustic phonons. In this section we derive the connection between the coherent and conventional forms of Lattice-Dynamical (piezoelectric) Quantum 1/f Effect (LD-Q1/fE) similarly to the way this was done in 1985 for the usual QED-Q1/fE [3].

In QLD, the amplitude of the piezo-elastic transversal lattice mode with wave vector q is the Fourier component q of the displacement caused by an electron in the lattice, expanded over a cubic box of volume V

$$A_{\vec{q}} = \frac{-iv_s(2\pi g / q^3 V)^{1/2}}{v_s - \hbar \hat{q} \cdot \vec{k} / m} \tag{13}$$

This is similar to Eq. (5). The part corresponding to the motion is obtained by subtracting the static part (Eq. 13 for k=0) that applies for an electron at rest

$$A^{mot}_{\vec{q}} = -iv_s\left(\frac{2\pi g}{q^3 V}\right)^{\frac{1}{2}}\left[\frac{1}{v_s - \hbar\hat{q}\cdot\vec{k}/m} - \frac{1}{v_s}\right] = -iv_s\left(\frac{2\pi g}{q^3 V}\right)^{\frac{1}{2}}\frac{\hbar\hat{q}\cdot\vec{k}}{mv_s(v_s - \hbar\hat{q}\cdot\vec{k}/m)} \tag{14}$$

The corresponding motional energy W^{mot} for an electron of wave vector k is thus

$$v_s^{-2}W^{mot}_{\vec{k}} = \sum_{\vec{q}} |A^{mot}_{\vec{q}}|^2\,\hbar\omega_q = \frac{2\pi g}{V}\frac{\hbar^3}{m^2 v_s}\frac{V}{(2\pi)^3}\int d\Omega\int_0^{q_D} dq \frac{k^2\cos^2\theta}{(v_s - \hbar k\cos\theta/m)^2} \tag{15}$$

Here q_D is the Debye wave vector. The integration yields

$$W^{mot}_{\vec{k}} = \frac{g\hbar^3}{2\pi m^2 v_s}\frac{V}{(2\pi)^3}\int_0^{q_D} dq\int_{-1}^{1}\frac{k^2x^2dx}{(1-\hbar kx/mv_s)^2} = \frac{g}{2\pi}\frac{mv_s^2 q_D}{k}F\left(\frac{v}{v_s}\right) \tag{16}$$

With $q_D=1/r'_0$, we obtain thus for cylindrical symmetry

$$W^{mot}_{\vec{k}} = \frac{g}{2\pi}\frac{mv_s^2}{kr'_o}F\left(\frac{v}{v_s}\right)\xrightarrow{\text{Cylindrical Symm}}\frac{g}{2\pi}\frac{mv_s^2}{kr'_o}F\left(\frac{v}{v_s}\right)N'\ln\frac{R}{r'_0} \tag{17}$$

N' is again the number of carriers per unit length of the sample in the direction of current flow, and *m** is the electron effective mass. Here we introduced the function

$$F(u/v_s) = 2(u/v_s)\{1 + 1/[1 - (u/v_s)^2]\} + \ln[1-(u/v_s)]^2 - \ln[1+(u/v_s)]^2. \tag{18}$$

We introduce the ratio of the coherent piezoelectric energy of the drift motion divided by the additional kinetic energy of the carriers introduced by the drift velocity u.

$$s' = \frac{\frac{g}{2\pi}\frac{mv_s^2}{kr'_o}F\left(\frac{u}{v_s}\right)N'\ln\frac{R}{r'_0}}{N'\,mu^2/2} = \frac{g}{\pi}\frac{v_s^2}{u^2kr'_o}F\left(\frac{u}{v_s}\right)\ln\frac{R}{r'_0} \tag{19}$$

This is the desired ratio s' of terms present in the hamiltonian, corresponding in QLD to the parameter s.

As before, the coherent and conventional forms of QLD-Q1/fE are superposed with weights s/(1+s) and 1/(1+s). For the intermediate case, i.e., for sizes in between coherent and incoherent, we have $S_j(f)/j^2 = \alpha/Nf$ with α defined as

$$\alpha = \alpha_{incoh}\frac{1}{(1+s)} + \alpha_{coh}\frac{s}{(1+s)} \tag{20}$$

Again, similar to s, the weight s' is the ratio between the piezoelectric or LD form of kinetic energy [associated with the electric current present in the sample or device, that is coherent], and the incoherent kinetic energy $m_{eff}u^2/2$ of individual particles, [caused by the drift velocity u].

The function F present in the definition of s' is given by Eq. (18) as

$$F(u/v_s) = 2(u/v_s)\{1+1/[1-(u/v_s)^2]\} + \ln[1-(u/v_s)]^2 - \ln[1+(u/v_s)]^2. \tag{21}$$

For $u/v_s \ll 1$, $F(u/v_s) = (2/3)(u/v_s)^3$; then *s*' can be simplified as $s' = 2gN'\hbar/3\pi m^* v_s$. This expression can be used for small drift velocity, much smaller than the sound velocity in the semiconductor. For $1-(u/v_s) << 1$, $F(u/v_s) = 1/[1-(u/v_s)]$, then s' can be approximated as $s' = (gN'\_/\pi m^* v_s)(v_s/u)^3\{1/[1-(u/v_s)]\}$, which is very large. This expression is used for drift velocities approaching the speed of sound.

5. Subharmonics

It is well known that quantum 1/f noise arises from a k-space integral over an integrand proportional to $1/k^3$. A similar dependence proportional to $1/\lambda^3$ is possible in the cubic lattice of subharmonics, because of the antenna factor of the elastic coupling of a small crystal to its support. Subharmonics form a cubic lattice like the k lattice turned inside out. The number of modes between λ and $\lambda+d\lambda$ is $4\pi\lambda^2 d\lambda$. This would lead to a low-frequency component of 1/f noise, generated in nonlinear sytems by the infinite set of subharmonic modes.

6. Conclusions

For lattice-dynamical (piezoelectric Q1/f noise based on TA infraquanta) the weight parameter, or coherence parameter, s' has the same role as the parameter s had for the usual (QED) Q1/f noise. It gives a rough interpolation approximation of the piezoelectric Q1/f coefficient by combining the conventional and coherent parts in a weighted sum

$$\boxed{\alpha_H = fNS_{\delta j/j}(f) = [1/(1+s')](4g/3\pi)(\Delta\mathbf{k}/k_S)^2 + [s'/(1+s')]\{2g/\pi[1-(<\mathbf{k}>/k_S)^2]\},} \quad (22)$$

where v_S is the speed of sound, $<\mathbf{k}>=m^*<\mathbf{v}>/\hbar$, $|<\mathbf{v}>| = u$ is the drift velocity of the carriers, $\Delta\mathbf{k}/k_S=\Delta\mathbf{v}/v_S$, and

$$\boxed{k_S = m^* v_S/\hbar; \quad s' = (gN'\hbar/\pi m^* v_S)(v_S/u)^3 F(u/v_S).} \quad (23)$$

Here N' is the number of carriers per unit length of the sample in the direction of current flow, and

$$\boxed{F(u/v_S) = 2(u/v_S)\{1 + 1/[1 - (u/v_S)^2]\} + \ln[1-(u/v_S)]^2 - \ln[1+(u/v_S)]^2.} \quad (24)$$

The coherence parameter s' was calculated in the same way in which s was introduced in the QED case for the connection between coherent and conventional Q1/f). It is the ratio of the coherent piezoelectric energy of the drift motion divided by the additional kinetic energy of the carriers introduced by the drift velocity u.

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