The nature of fundamental 1/f Noise in quantum sensing technology, negative entropy and uncertainty principle

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ABSTRACT

Most sensors traduce various environmental, industrial, or laboratory parameters into electrical signals. Quantum sensing pushes this to the ultimate limit of achievable sensitivity, detectivity, resolution, and stability. Quantum mechanics sets these limits through the uncertainty principle. It causes the ubiquitous 1/f noise, as we show here for $LiNbO_3$.

Quantum 1/f theory, known as the quantum theory of the fundamental 1/f noise phenomenon, is a new aspect of quantum mechanics, that governs the nonlinear interaction of particle and field, introducing the new notions of "physical cross sections" (PCS) and "physical process rates" (PPR). These contain the fundamental macroscopic quantum 1/f fluctuations. They yield the usually defined PCS and PPR with infrared radiative corrections, when the expectation value is taken over the low-frequency photon states of negative entropy.

The physical charged particle, such as an electron, contains both the bare particle and its electromagnetic field. The latter is in a coherent state, where the phase of the electromagnetic field oscillators is determined. Heisenberg's uncertainty principle then requires that the energy must be uncertain, leading to nonstationary states describing the particle. The particle's resulting fractional current fluctuations in large devices always have the spectrum $2\alpha/\pi f$, where $\alpha=1/137$ is the fine structure constant.

Keywords: Quantum 1/f Noise, Sensors, Spintronics, Infrared detectors, Entropy, Nanodevices

I. INTRODUCTION

The conventional Quantum 1/f Effect (Q1/fE[1-25]) is a fundamental fluctuation of all currents of charged particles j, of all scattering cross sections σ_s, recombination cross sections σ_r, tunneling rates or transition rates of any kind Γ, in short of all physical cross sections σ and process rates Γ, given by the universal formula

$$S(f) = 2\alpha A/fN \tag{1}$$

(conventional quantum 1/f equation[1-18]) for small devices, and

$$S(f) = 2\alpha/\pi fN \tag{2}$$

(coherent quantum 1/f equation[19, 16-18]) for large devices. These two Q1/fE forms can be combined into a single general formula[20, 16-18], as we show at the end of this paper. Here S(f) is the spectral density of fractional fluctuations in current, $\delta j/j$, in scattering or recombination cross section $\delta\sigma/\sigma$, or in any other process rate $\delta\Gamma/\Gamma$. $\alpha=e^2/\hbar c=1/137$ is Sommerfeld's fine structure constant, a magic number of our world depending only on Planck's constant $\hbar$, the charge of the electron e and the speed of light in vacuum c. $A=2(\Delta\mathbf{v}/c)^2/3\pi$ is essentially the square of the vector velocity change $\Delta\mathbf{v}$ of the scattered particles in the scattering process whose fluctuations we are considering, in units of c. Finally, N is the number of particles used to define the notion of current j, of cross section σ or of process rate Γ.

Due to the presence of a bremsstrahlung emission amplitude in scattering, the scattered current per unit incoming flux, also called by us "the physical scattering cross section σ" will contain several energy and momentum components. These interfere with each other, causing conventional quantum 1/f noise, i.e., quantum fluctuations of σ with a 1/f spectral density. The quantum expectation $\langle\sigma\rangle$ of σ gives the usual definition of the cross section. The quantum fluctuations are macroscopic at low frequencies because of the 1/f dependence, and because they do not have the fast Zitterbewegung variation which is usually associated with quantum fluctuations.

Another manifestation, or aspect, of the same Q1/fE phenomenon is the uncertainty of the mass shell of a charged particle due to the infinite range of the Coulomb interaction[22-25]. This requires a new picture[22-25] in which the long range part of the interaction is included in the unperturbed hamiltonian. This, in turn, leads for the first time to the existence of the interaction representation, and to finite matrix elements. It also leads to a branch-point singularity in the propagator, to a more complex free particle notion, to a coherent state of the field associated with the free particle. As we prove below, however, it also leads to the presence of macroscopic coherent quantum 1/f fluctuations of all charged currents (coherent quantum 1/f effect, as this author calls it).

How is it possible to obtain the additional entropy implied in the 2 forms, or aspects, of the quantum 1/f effect? All we have is unitary evolution of the quantum state of the particles, of the electromagnetic field of the universe, as well as of a potential observer. Entropy is known to be conserved in this unitary evolution of such a pure state. The quantum information theory[26] approach allows us to understand this by introducing the notion of negative entropy as we show below.

2. SCHEMATIC DERIVATION OF THE CONVENTIONAL QUANTUM 1/f EFFECT

We start here first with a schematic derivation that clarifies the essential elements. Let's simplify our world by assuming that only one electromagnetic mode of the universe would exist, with frequency ω and wave vector k. Consider the field mode in its ground state. Also consider two incoming identical charged particles with the same well-defined wave vector, both being scattered by some potential. Both the initial (incoming) and the final (scattered) state represent a pure state. The initial state is $|{+}{+}{-}\rangle_o$, where we reserved the first two arguments (+) of the ket for the two electrons still having their full energy, and the last (-) for the field in its ground state. Due to the interaction of the electrons with the field, the final state will be in first approximation

$$|f\rangle = (|{+}{+}{-}\rangle + \gamma|{+}{-}{+}\rangle/\sqrt{2} + \gamma|{-}{+}{+}\rangle/\sqrt{2})/(\sqrt{1+\gamma^2}), \qquad (3)$$

where γ is the emission amplitude of a bremsstrahlung photon, i.e., for the excitation of the field oscillator from its ground state (-) to its first excited state (+). The first two arguments label the state of the charged particles, indicating the presence of an energy loss with (-) and the persistence in the same energy state with (+). The third argument of the kets always labels the field oscillator, as mentioned.

The corresponding density operator of the pure state obtained is $\rho = |f\rangle\langle f|$. Its quantum von Neumann entropy $S/k = \sigma = -\mathrm{Tr}(\rho \log \rho)$ is zero.

2.1 Paradoxical entropy increase in 1/f noise

Ignoring the field oscillator, i.e., taking the trace of over the field oscillator label, we obtain a classically correlated system, a mixture of two pure states, described by the density operator

$$[|{+}{+}\rangle\langle{+}{+}| + \gamma^2(|{+}{-}\rangle + |{-}{+}\rangle)(\langle{+}{-}| + \langle{-}{+}|)/2]/(1+\gamma^2). \qquad (4)$$

The second term gives the Q1/fE at $f=\omega/2\pi$, as we show below. The corresponding entropy is

$$\log(1+\gamma^2) - (\gamma^2\log\gamma^2)/(1+\gamma^2) > 0 \text{ for } 0\le\gamma^2\le 1.$$

The entropy of the system thus appears to have increased although according to quantum mechanics it can not increase. Indeed, according to quantum mechanics, time evolution of a state occurs through a unitary transformation. The latter, however, is known to leave $\sigma = -\mathrm{Tr}(\rho \log \rho)$ invariant. Due to the known $f^{-1/2}$ dependence of γ, *this yields 1/f noise* when all electromagnetic oscillators are switched on, as we show in detail in Sec. 4 below. This is our schematic derivation.

2.2 Quantum information theory

The Quantum Information Theory[26] solves this paradox. When two systems A and B with quantum von Neumann entropy $\sigma(A) = -Tr_A(\rho_A \log\rho_A)$ and $\sigma(B) = -Tr_B(\rho_B \log\rho_B)$ form a composite system AB of entropy $\sigma(AB) = -Tr_{AB}(\rho_{AB}\log\rho_{AB})$, we can prove that we have to write

$$\sigma(AB) = \sigma(A) + \sigma(B|A) = \sigma(B) + \sigma(A|B), \tag{5}$$

in perfect analogy with classical entropies or information. We have introduced $\sigma(A|B)$ as the von Neumann entropy of A conditional on B, or the entropy of A when we know B:

$$\sigma(A|B) = -Tr_{AB}[\rho_{AB}\log\rho_{A|B}]. \tag{6}$$

Here we have introduced the conditional density matrix $\rho_{A|B} = \rho_{AB}(1_A \otimes \rho_B)^{-1}$, the quantum analog similar to the classical conditional probability, where $\otimes$ is the tensor product in the joint Hilbert space and $\rho_B = Tr_A(\rho_{AB})$ is the marginal density matrix obtained by taking a partial trace over the variables associated with A.

3. NEGATIVE ENTROPY

The conditional entropy is usually negative in quantum entangled states. Finally, we introduce the quantum mutual entropy which represents the shared entropy, corresponding to the mutual information between A and B:

$$\sigma(A{:}B) = -Tr[\rho_{AB}\log\rho_{A:B}] \equiv \sigma(A) + \sigma(B) - \sigma(AB), \tag{7}$$

where

$$\rho_{A:B} = \rho_{AB}(\rho_A \otimes \rho_B)^{-1}. \tag{8}$$

Taking all logarithms in the base of 2, the entropies will be expressed in bites. Considering our paradoxical case discussed above for simplicity with $\gamma^2=1$, we obtain the following entropy diagram of our quantum mechanically entangled triplet of three systems comprising the charged particles A and B, as well as the field oscillator C:

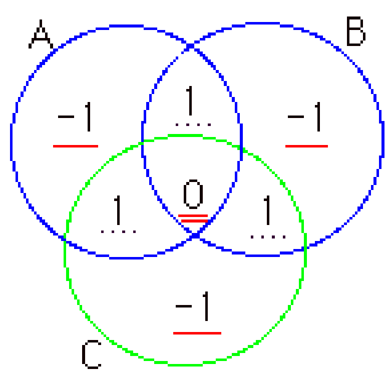

Figure 1: Entropy diagram of the quantum mechanically entangled triplet comprising the charged particles A and B, as well as the field oscillator C.

Here the quantum conditional entropies are underlined for emphasis, and the mutual quantum entropy has double underlining. Dotted underlining marks the mutual entropy of two of the subsystems, which is, however, not the mutual entropy of the whole system. We see that A, B and C have each 1 bit of entropy, as we expect, since each can be in a + or - state. In addition, $\sigma_{ABC} = 0$ as expected for a pure state. The negative conditional entropies indicate that this state is a pure quantum state which can not be obtained in classical physics.

If we ignore C by tracing over it, we obtain the classically correlated system AB with its positive entropy of 1 *and* the negative quantum entropy state of the ignored field oscillator, conditional on AB:

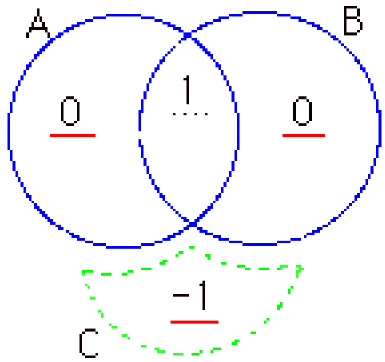

Figure 2: By tracing over C, we obtain the classically correlated system AB with its positive entropy of 1 and the negative quantum entropy state of the ignored field oscillator, conditional on AB.

3.1 Solution of the paradox

In conclusion, the negative conditional entropy of the ignored field oscillator compensates the positive entropy of the system of two particles. Unitarity and entropy conservation are both satisfied. This is the solution of the paradox. It explains why our earlier explanation of the Q1/fE in terms of a two-particle wave function was correct, in spite of the apparent lack of unitarity.

In the rest of the present paper we apply the scheme developed here to the real world with all field oscillators present in $|{+}{+}{-}\rangle_o$ and with N incident particles in $|{+}{+}{-}\rangle_o$.

4. DERIVATION OF THE CONVENTIONAL QUANTUM 1/F EFFECT

We start with the expression of the Heisenberg representation state $|f\rangle$ of N identical bosons of mass M emerging at an angle θ from some scattering process with undetermined bremsstrahlung energy losses reflected in their one-particle waves $\varphi_i(\boldsymbol{\xi}_i)$

$$|f\rangle = (N!)^{-1/2}\, \Pi_i \int d^3\xi_i \varphi_i(\boldsymbol{\xi}_i)\psi^+(\boldsymbol{\xi}_i)\, |0\rangle = \Pi_i \int d^3\xi_i \varphi_i(\boldsymbol{\xi}_i)\, |f^o\rangle, \tag{9}$$

where $\psi^+(\boldsymbol{\xi}_i)$ is the field operator creating a boson with position vector $\boldsymbol{\xi}_i$ and $|0\rangle$ is the vacuum state, while $|f^o\rangle$ is the vacuum field state with N bosons of position vectors $\boldsymbol{\xi}_i$ with i = 1...N. All products and sums in this paper run from 1 to N, unless otherwise stated.

To calculate the particle density autocorrelation function in the outgoing scattered wave, we need the expectation value of the operator

$$O(\mathbf{x}_1,\mathbf{x}_2) = \psi^+(\mathbf{x}_1)\psi^+(\mathbf{x}_2)\psi(\mathbf{x}_2)\psi(\mathbf{x}_1), \tag{10}$$

known as the operator of the pair correlation. Using the commutation properties of the boson field operators, we first calculate the matrix element

$$N!\langle S^o|O|f^o\rangle = \Sigma'_{\mu\nu}\Sigma'_{mn}\delta(\boldsymbol{\eta}_\nu-\mathbf{x}_1)\delta(\boldsymbol{\eta}_\mu-\mathbf{x}_2)\delta(\boldsymbol{\xi}_n-\mathbf{x}_1)\delta(\boldsymbol{\xi}_m-\mathbf{x}_2)\Sigma_{(i,j)}\Pi'_{ij}\delta(\boldsymbol{\eta}_j-\boldsymbol{\xi}_i). \tag{11}$$

Here the prime excludes $\mu=\nu$ and m=n in the summations and excludes i=m, i=n, $j=\mu$ and $j=\nu$ in the product. The summation $\Sigma_{(i,j)}$ runs over all permutations of the remaining N-2 values of i and j. On this basis we now calculate the complete matrix element

$$\langle f|O|f\rangle = [1/N(N-1)]\, \Sigma'_{\mu\nu}\Sigma'_{mn}\int d^3\eta_\mu \int d^3\eta_\nu \int d^3\xi_m \int d^3\xi_n$$

$$\varphi_\mu^*(\eta_\mu)\varphi_\nu^*(\eta_\nu)\varphi_m(\xi_m)\varphi_n(\xi_n)\delta(\eta_\nu-\mathbf{x}_1)\delta(\eta_\mu-\mathbf{x}_2)\delta(\xi_n-\mathbf{x}_1)\delta(\xi_m-\mathbf{x}_2). \quad (12)$$
$$= [1/N(N-1)]\ \Sigma'_{\mu\nu}\Sigma'_{mn}\ \varphi_\mu^*(\mathbf{x}_2)\varphi_\nu^*(\mathbf{x}_1)\varphi_m(\mathbf{x}_1)\varphi_n(\mathbf{x}_2)$$

The one-particle states are spherical waves emerging from the scattering center located at x=0:

$$\varphi(\mathbf{x}) = (C/x)e^{iKx}\ [1 + \Sigma_{\mathbf{k}l}b(\mathbf{k},l)e^{-iqx}\ a^+_{\mathbf{k},l}]. \quad (13)$$

Here C is an amplitude factor, K the boson wave vector magnitude, $\gamma = b(\mathbf{k},l)$ the bremsstrahlung amplitude for photons of wave vector $\mathbf{k}$ and polarization l, while $a^+_{\mathbf{k},l}$ is the corresponding photon creation operator, allowing the photon state to be created from the vacuum if Eq. (13) is inserted into Eq. (8). The momentum magnitude loss $q=Mck/hK\equiv Mf/hK$ is necessary for energy conservation in the Bremsstrahlung process. Substituting Eq, (13) into Eq. (12), we obtain

$$\langle f|O|f\rangle = |C/x|^4\{N(N-1) + 2(N-1)\Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2[1+\cos q(x_1-x_2)]\}, \quad (14)$$

where we neglected a small term of higher order in $b(\mathbf{k},l)$. To perform the angular part of the summation in Eq. (14), we calculate the current expectation value of the state in Eq. (13), and compare it to the well-known cross section without and with bremsstrahlung

$$\mathbf{j} = (h\mathbf{K}/Mx^2)[1 + \Sigma_{kl}|b(k,l)|^2] = \mathbf{j}_o[1 + \int\alpha A df/f], \quad (15)$$

where the quantum fluctuations have disappeared, where $\alpha=e^2/hc$ is the fine structure constant, $\alpha A=(2\alpha/3\pi)(\Delta\mathbf{v}/c)^2$ is the fractional bremsstrahlung rate coefficient, also known in QED as the infrared exponent, and the 1/f dependence of the bremsstrahlung part displays the well-known infrared catastrophe, i.e., the emission of a logarithmically divergent number of photons in the low frequency limit. Here $\Delta\mathbf{v}$ is the velocity change $h(\mathbf{K}-\mathbf{K}_o)/M$ of the scattered boson, and $f=ck/2\pi$ the photon frequency. Eq. (14) gives

$$\langle f|O|f\rangle = |C/x|^4\{N(N-1) + 2(N-1)(\int\alpha A df/f)[1+\cos q(x_1-x_2)]\}, \quad (16)$$

which is the pair correlation function, or density autocorrelation function along the scattered beam with df/f=dq/q. The spatial distribution fluctuations along the scattered beam will also be observed as fluctuations in time at the detector, at any frequency f. According to the Wiener-Khintchine theorem, we obtain the spectral density of fractional scattered particle density ρ, (or current j, or cross section σ) fluctuations in frequency f or wave number q by dividing the coefficient of the cosine by the constant term N(N-1):

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/fN,\ [\text{or } =2\alpha A/f(N-1) \text{ for fermions}], \quad (17)$$

where N is the number of particles or current carriers used to define the current j whose fluctuations we are studying. Quantum 1/f noise is thus a fundamental 1/N effect. A more physical derivation is in a companion paper in this volume.

The exact value of the exponent of f in Eq. (17) can be determined by including the contributions from all real and virtual multiphoton processes of any order, and turns out to be αA-1, rather than -1, which is important only philosophically, since αA«1. The spectral integral is thus convergent at f=0.

5. PHYSICAL DERIVATION OF THE COHERENT QUANTUM 1/F EFFECT

This effect arises in a beam of electrons (or other charged particles propagating freely in vacuum) from the definition of the physical electron as a bare particle plus a coherent state of the electromagnetic field. It is caused by the energy spread characterizing any coherent state of the electromagnetic field oscillators, an energy spread which spells non-stationarity, i.e., fluctuations. To find the spectral density of these inescapable fluctuations which are known to characterize any quantum state which is not an energy eigenstate, we use an elementary physical derivation based on Schrödinger's definition of coherent states, followed by a rigorous derivation from a well-known quantum-electrodynamical propagator. The quantum chaotic character of these fluctuations is caused by the fact that they are inherent in the quantum dynamics of the system, and are not caused by the introduction of any external random element as would be the case in stochastic fluctuations.

The coherent quantum 1/f effect will be derived in three steps. First we consider a hypothetical world with just one single mode of the electromagnetic field coupled to a beam of charged particles. Considering the mode to be in a coherent state, we calculate the autocorrelation function of the quantum fluctuations in the particle-density (or concentration) which arise from the nonstationarity of the coherent state. Second, we calculate the amplitude with which this one mode is represented in the field of an electron, according to electrodynamics. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the electromagnetic field be characterized by the wave vector q, the angular frequency $\omega = cq$ and the polarization λ. Denoting the variables q and λ simply by q in the labels of the states, we write the coherent state[22-23] of amplitude $|z_q|$ and phase $\arg z_q$ in the form

$$|z_q\rangle = \exp[-(1/2)|z_q|^2] \exp[z_q a_q^+] |0\rangle = \exp[-(1/2)|z_q|^2] \sum_{n=0} (z_q^n) /n! \, |n\rangle. \quad (19)$$

Here a_q^+ is the creation operator which adds one energy quantum to the energy of the mode. Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n\rangle = (2^n n! \sqrt{\pi})^{-1/2} \exp[-x^2/2] \, H_n(x) \, e^{in\omega t}. \quad (20)$$

This yields for the coherent state $|z_q\rangle$ the representation

$$\Psi_q(x) = \exp[-(1/2)|z_q|^2]\exp[-x^2/2] \sum_{n=0} \{[z_q e^{i\omega t}]^n/[n!(2^n\sqrt{\omega})]^{1/2}\} H_n(x)$$

$$= \exp[-(1/2)|z_q|^2]\exp[-x^2/2]\exp[-z_q^2 e^{-2i\omega t} + 2xz_q e^{i\omega t}]. \quad (21)$$

In the last form the generating function of the Hermite polynomials was used. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time t or the phase of z_q, is, for $|z_q| \ll 1$,

$$P_q(\tau,x) = \langle |\Psi_q|_t^2 \mid \Psi_q|_{t+\tau}^2 \rangle = \{1 + 8x^2|z_q|^2[1 + \cos \omega\tau] - 2|z_q|^2\}\exp[-x^2/2]. \quad (22)$$

Integrating over x from $-\infty$ to ∞, we find the autocorrelation function

$$A^1(\tau) = (2)^{-1/2}\{1 + 2|z_q|^2\cos \omega\tau\}. \quad (23)$$

This result shows that the probability distribution contains a constant background with small superposed oscillations of frequency ω. Physically, the small oscillations in the total probability describe self-organization or bunching of the particles in the beam. They are thus more likely to be found in a measurement at a certain time and place than at other times and places relative to each other along the beam. Note that for $z_q = 0$ the coherent state becomes the ground state of the oscillator which is also an energy eigenstate, and therefore stationary and free of oscillations.

We now determine the amplitude z_q with which the field mode q is represented in the physical electron. One way to do this is to let a bare particle dress itself through its interaction with the electromagnetic field, i.e. by performing first order perturbation theory with the interaction Hamiltonian

$$H' = A_\mu j^\mu = -(e/c)\mathbf{v}\cdot\mathbf{A} + e\phi \, , \quad (24)$$

where $\mathbf{A}$ is the vector potential and ϕ the scalar electric potential. Another way is to Fourier expand the electric potential e/r of a charged particle in a box of volume V. In both ways we obtain the well-known result

$$|z_q|^2 = \pi(e/q)^2(\hbar cqV)^{-1}. \quad (25)$$

Considering now all modes of the electromagnetic field, we obtain from the single - mode result of Eq. (23)

$$A(\tau) = C\, \Pi_q\{1 + 2|z_q|^2\cos \omega_q\tau\} = C\{1 + \Sigma_q 2|z_q|^2\cos \omega_q\tau\}$$

$$= C\{1 + 4(V/2^3\pi^3) \int d^3q\, |z_q|^2 \cos \omega_q\tau\} \quad (26)$$

Here we have again used the smallness of z_q and we have introduced a constant C. Using Eq. (25) we obtain

$$A(\tau) = C\{1 + 4\pi(V/2^3\pi^3)(4\pi/V)(e^2/\hbar c) \int (dq/q) \cos \omega_q\tau\}$$

$$= C\{1 + 2(\alpha/\pi) \int \cos(\omega\tau) d\omega/\omega\}. \quad (27)$$

Here $\alpha = e^2/\hbar c$ is Sommerfeld's fine structure constant 1/137. The first term in curly brackets is unity and represents the constant background, or the d.c. part of the current carried by the beam of particles through vacuum. The autocorrelation function for the relative (fractional) density fluctuations, or for the current density fluctuations in the beam of charged particles is obtained therefore by dividing the second term in curly brackets by the first term. The constant C drops out when the fractional fluctuations are considered. According to the Wiener-Khintchine theorem, the coefficient of $\cos\omega\tau$ is the spectral density of the fluctuations, $S_{|\psi|^2}$ for the particle concentration, or S_j for the current density $j = e(k/m)|\psi|^2$

$$S_{|\psi|^2}\langle|\psi|^{-2}\rangle = S_j\langle j\rangle^{-2} = 2(\alpha/\pi f N) = 4.6{\cdot}10^{-3}\, f^{-1}N^{-1}. \quad (28)$$

Here we have included the total number N of charged particles which are observed simultaneously in the denominator, because the noise contributions from each particle are independent. This result is related to the conventional Quantum 1/f Effect considered in the next section. A similar calculation yields the gravidynamical quantum 1/f effect (QGD 1/f effect) by substituting gravitons for the photons considered so far as infraquanta.

6. RIGOROUS DERIVATION OF THE COHERENT QUANTUM 1/F EFFECT

The coherent quantum 1/f effect is a quantum fluctuation effect present in any extended current due to the definition of the physical electron as a system composed of the bare particle plus its electomagnetic field which is in a coherent state and has therefore indefinite energy. This in turn causes the state to be non-stationary. The non-stationarity is expressed by the coherent quantum 1/f effect. An elementary physical derivation is given in Sec. VII.

The present derivation is based on the well-known new propagator $G_s(x'-x)$ derived relativistically[22-25] in 1975 in a new picture required by the infinite range of the Coulomb potential. We use the nonrelativistic form[28]

$$-i\langle\Phi_o|T\psi_{s'}(x')\psi_s^{+}(x)\ \ |\Phi_o\rangle \equiv \delta_{ss'}\, G_s(x'-x)$$

$$= (i/V)\sum_{\mathbf{p}}\{\ \exp i[\mathbf{p}(\mathbf{r}-\mathbf{r}')-\mathbf{p}^2(t-t')/2m]/\hbar\ \}n_{\mathbf{p},s}\{-i\mathbf{p}(\mathbf{r}-\mathbf{r}')/\hbar\ \ +i(m^2c^2+\mathbf{p}^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}. \quad (29)$$

Here $\alpha=e^2/\hbar c=1/137$ is Sommerfeld's fine structure constant, $n_{\mathbf{p},s}$ the number of electrons in the state of momentum $\mathbf{p}$ and spin s, m the rest mass of the fermions, $\delta_{ss'}$ the Kronecker symbol, c the speed of light, $x=(\mathbf{r},t)$ any space-time point and V the volume of a normalization box. T is the time-ordering operator which orders the operators in the order of decreasing times from left to right and multiplies the result by $(-1)^P$, where P is the parity of the permutation required to achieve this order. For equal times, T normal-orders the operators, i.e., for t=t' the left-hand side of Eq. (26) is $i\langle\Phi_o|\psi_s^{+}(x)\ \ \psi_{s'}(x')|\Phi_o\rangle$. The state Φ_o of the N electrons is described by a Slater determinant of single-particle orbitals.

The resulting spectral density coincides with the result $2\alpha/\pi f N$, first derived[19] directly from the coherent state of the electromagnetic field of a physical charged particle. The connection with the conventional quantum 1/f effect was suggested later[20].

To calculate the current autocorrelation function we need the density correlation function, which is also known as the two-particle correlation function. The two-particle correlation function is defined by

$$\langle\Phi_o|T\psi_s^{+}(x)\ \ \psi_s(x)\psi_{s'}^{+}(x')\ \ \psi_{s'}(x')|\Phi_o\rangle =$$

$$\langle\Phi_o|\psi_s^{+}(x)\ \ \psi_s(x)|\Phi_o\rangle\langle\Phi_o|\psi_{s'}^{+}(x')\ \ \psi_{s'}(x')|\Phi_o\rangle - \langle\Phi_o|T\psi_{s'}(x')\psi_s^{+}(x)\ \ |\Phi_o\rangle\langle\Phi_o|T\psi_s(x)\psi_{s'}^{+}(x')\ \ |\Phi_o\rangle. \quad (30)$$

The first term can be expressed in terms of the particle density of spin s, $n/2 = N/2V = <\Phi_o|\psi_s^+(x)\ \ \psi_s(x)|\Phi_o>$, while the second term can be expressed in terms of the Green function (1) in the form

$$A_{ss'}(x-x') \equiv <\Phi_o|\psi_s^+(x)\ \ \psi_{s'}^+(x')\ \ \psi_{s'}(x')\psi_s(x)|\Phi_o> = (n/2)^2 + \delta_{ss'}\, G_s(x'-x)G_s(x-x'). \tag{31}$$

The "relative" autocorrelation function A(x-x') describing the normalized pair correlation independent of spin is obtained by dividing by n^2 and summing over s and s'

$$A(x-x') = 1 - (1/n^2)\sum_s G_s(x-x')G_s(x'-x) = 1 - (1/N^2)\sum_{s\mathbf{pp'}}\{\ \exp i[(\mathbf{p}-\mathbf{p'})(\mathbf{r}-\mathbf{r'})-(\mathbf{p}^2-\mathbf{p'}^2)(t-t')/2m]/\hbar\ \}n_{\mathbf{p},s}n_{\mathbf{p'},s}$$
$$\times\{\mathbf{p}(\mathbf{r}-\mathbf{r'})/\hbar - (m^2c^2+\mathbf{p}^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}\ \{\mathbf{p'}(\mathbf{r}-\mathbf{r'})/\hbar - (m^2c^2+\mathbf{p'}^2)^{1/2}(t-t')(c/\hbar)\}^{\alpha/\pi}. \tag{32}$$

Here we have used Eq. (29). We now consider a beam of charged fermions, e.g., electrons, represented in momentum space by a sphere of radius p_F, centered on the momentum $\mathbf{p}_o$ which is the average momentum of the fermions. The energy and momentum differences between terms of different **p** are large, leading to rapid oscillations in space and time which contain only high-frequency quantum fluctuations. The low-frequency and low-wavenumber part A_l of this relative density autocorrelation function is given by the terms with $\mathbf{p}=\mathbf{p'}$

$$A_l(x-x') = 1 - (1/N^2)\sum_s\sum_{\mathbf{p}} n_{\mathbf{p},s}\ \{\mathbf{p}(\mathbf{r}-\mathbf{r'})/\hbar - (m^2c^2+\mathbf{p}^2)^{1/2}(t-t')(c/\hbar)\}^{2\alpha/\pi} \tag{33}$$

$$\approx 1 - (1/N)|\mathbf{p}_o(\mathbf{r}-\mathbf{r'})/\hbar - mc^2\tau/\hbar|^{2\alpha/\pi}\ \text{ for }\ p_F << |p_{o3} - mc^2\tau/z|. \tag{34}$$

Here we have used the mean value theorem, considering the $2\alpha/\pi$ power as a slowly varying function of **p** and neglecting $\mathbf{p}_o$ in the coefficient of $\tau \equiv t-t'$, with $z \equiv |\mathbf{r}-\mathbf{r'}|$. Using the identity[27], with arbitrarily small cutoff ω_o, we obtain from Eq. (34) with $\theta \equiv |\mathbf{p}_o(\mathbf{r}-\mathbf{r'})/\hbar - mc^2\tau/\hbar\,|$ the exact form

$$A_l(x-x') = 1 + [(2\alpha/\pi N)\int_{\omega_o}(mc^2/\hbar\omega)^{2\alpha/\pi}\cos(\theta\omega)d\omega/\omega]\{\cos\alpha + (2\alpha/\pi)\sum_{n=0}(\theta\omega_o)^{2n-2\alpha/\pi}[(2n)!(2n-2\alpha/\pi)]^{-1}\}^{-1}. \tag{35}$$

This indicates a $\omega^{-1-2\alpha/\pi}$ spectrum and a 1/N dependence of the spectrum of fractional fluctuations in density n and current j, if we neglect the curly bracket in the denominator which is very close to unity for very small ω_o. The fractional autocorrelation of current fluctuations δj is obtained by multiplying Eq. (35) on both sides with ep_o/m, and dividing by $(enp_o/m)^2$ which is the square of the average current density j, instead of just dividing by n^2. It is the same as the fractional autocorrelation for quantum density fluctuations. Then Eq. (35) for the coherent Quantum Electrodynamical chaos process in electric currents can be written also in the form

$$S_{\delta j/j}(k) \approx [2\alpha/\pi\omega N][mc^2/\hbar\,\omega]^{2\alpha/\pi} \approx 2\alpha/\pi\omega N = 0.00465/\omega N. \tag{36}$$

Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations[4, 17]. Most of the conventional quantum 1/f fluctuations in physical cross sections and process rates are also mobility fluctuations, but some are also in the recombination speed or tunneling rate.

7. TRANSITION BETWEEN THE COHERENT AND INCOHERENT CASES

We turn now to the connection to the coherent Quantum 1/f Effect. The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy E_m, per unit length of the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int(B^2/8\pi)d^3x = [nevS/c]^2\ln(R/r) \quad (37)$$

is much smaller than the total kinetic energy E_k of the drift motion of the individual carriers

$$E_k = \sum mv^2/2 = nSmv^2/2 = E_m/s. \quad (38)$$

Here we have introduced the magnetic field B, the carrier concentration n, the cross sectional area S and radius r of the cylindrical sample (e.g., a current carrying wire), the radius R of the electric circuit, and the "coherence ratio"

$$s = E_m/E_k = 2ne^2S/mc^2\ln(R/r) \approx 2e^2N'/mc^2 = 2N'r_o, \quad (39)$$

where N' = nS is the number of carriers per unit length of the sample and the natural logarithm ln(R/r) has been approximated by one in the last forms. Here $r_o = 2.84\ 10^{-13}$ cm is the classical radius of the electron. In general, thus, s = $5.69{\cdot}10^{-13}$ cm $\times$N', i.e., the number of carriers in a very thin sample slice of about 5.7 fermi thickness. We expect the observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN] \quad (40)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side

$$\alpha_H = [1/(1+s)]\alpha_{Hconv} + [s/(1+s)]\alpha_{Hcoh}. \quad (41)$$

In these equations $\alpha A=2\alpha(\Delta\mathbf{v}/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum 1/f effect. These equations are limited to quantum 1/f mobility (or diffusion) fluctuations, and do not include the quantum 1/f noise in the surface and bulk recombination cross sections, in the surface and bulk trapping centers, in tunneling and injection processes, in emission or in transitions between two solids.

Note that when n = 10^{16} cm^{-3}, the coherence ratio s introduced here equals the unity for the critical value N' = N" = $2{\bullet}10^{12}$/cm., e.g. for a cross section S = $2{\bullet}10^{-4}$ cm^2 of the sample. For small samples with N'<<N" only the first term survives, while for N'>N" the second term in Eq. (41) is dominant.

The conventional and coherent Q1/fE was successfully applied to FETs, HFETs, BJTs, and HBTs, as well as to quartz resonators and amplifiers.

8. ELECTRO-OPTIC QUANTUM 1/F FLUCTUATIONS IN $LiNbO_3$ CRYSTALS

This approach will now be applied to the important ferroelectric $LiNbO_3$ which is used in integrated acousto-optic device modules (8), for instance in integrated optic RF power spectrum analyzers.

Here is how it works. The rate Γ of photon-interactions which remove phonons from the main $LiNbO_3$ resonator mode is modulated by the quantum 1/f effect. Therefore it exhibits observable quantum fluctuations, while its expectation value remains constant. Indeed, whenever a phonon is removed from the main resonator mode, the time-derivative of the polarization vector of the $LiNbO_3$ crystal $d\mathbf{P}/dt=\dot{\mathbf{P}}$, is suddenly affected, suffering a step-like modification as a function of time. From Maxwell's equations we know, however, that $\dot{\mathbf{P}}$ is added to the current J and that such a modification of the current causes radiation. Solving Maxwell's equations we find that as a result of the phonon removal there is an energy of $(1/4\pi\varepsilon_o)^4(\Delta\dot{\mathbf{P}})^2/3c^3$ radiated away per unit frequency, i.e. per Hertz at any frequency f. Dividing this result by the energy of a photon hf, we find that there is thus a probability of $2\alpha(\Delta\dot{\mathbf{P}})^2/3\pi fe^2c^2$ for the emission (radiation) of a bremsstrahlung photon of frequency f. Here $\alpha = (1/4\pi\varepsilon_o)(2\pi e^2/hc) = 1/137$ is

Sommerfeld's fine structure constant, a dimensionless universal constant constructed from Planck's constant, the charge e of the electron, and the speed of light c. SI units are used here, while Gaussian units were used in (7).

Since there is a probability of $2\alpha(\Delta\dot{\mathbf{P}})^2/3\pi fe^2c^2 << 1$ for the emission of a photon of frequency f, the $LiNbO_3$ crystal suffers a reaction, or a recoil, in its quantum state, causing the phonon-emission rate Γ to perform quantum oscillations with frequency f and with two-sided spectral density S' of fractional fluctuations given by the same expression, $S'_{\delta\Gamma/\Gamma}(f) = 2\alpha(\Delta\dot{\mathbf{P}})^2/3\pi fe^2c^2$. This is the quantum 1/f effect. The one-sided spectrum is thus

$$S_{\delta\Gamma/\Gamma}(f) = 4\alpha(\Delta\dot{\mathbf{P}})^2/3\pi fe^2c^2. \tag{42}$$

This means that any radiation caused or implied by a quantum transition from one state to another comes with a price. It reacts back on the system, causing the rate of that transition to be modulated. It will thus exhibit observable macroscopic quantum fluctuations with a spectral density of fractional rate fluctuations identical to the photon emission probability accompanying the transition considered.

Let the resonator volume V be smaller than the phonon coherence length ε in all directions. In Eq. (42), $(\Delta\dot{\mathbf{P}})^2$ is the square of the dipole moment rate change associated with the process causing the removal of a phonon from the main oscillator mode: scattering of a main resonator mode phonon on a themal phonon of higher frequency $<\omega>\approx kT/\hbar$. To calculate $(\Delta\dot{\mathbf{P}})^2$, we write the energy W of the interacting mode $<\omega>$ in the form

$$W = n\hbar<\omega> = 2(Nm/2)(dx/dt)^2 = (Nm/e^2)(e\,dx/dt)^2 = (m/N)(e)^2(\dot{\mathbf{P}})^2 ; \tag{43}$$

The factor two includes the potential energy contribution. Here m is the reduced mass of the elementary oscillating dipoles, e their charge, g a polarization constant of the order of the unity, and N their number in the resonator. Applying a variation $\Delta n=1$ we get

$$\Delta n/n = 2|\Delta\dot{\mathbf{P}}|/|\dot{\mathbf{P}}|, \ \text{or} \ \Delta\dot{\mathbf{P}} = \dot{\mathbf{P}}/2n. \tag{44}$$

Solving Eq. (43) for $\dot{\mathbf{P}}$ and substituting into (44), we obtain

$$|\Delta\dot{\mathbf{P}}| = (N\hbar<\omega>/n)^{1/2}(e/2g) . \tag{45}$$

Substituting $\Delta\dot{\mathbf{P}}$ into Eq. (42), we get

$$\Gamma^{-2}S_{\Gamma}(f) = N\alpha\hbar<\omega>/3n\pi mc^2fg^2 \equiv \Lambda/f . \tag{46}$$

This result is applicable to the fluctuations in the loss rate Γ of the resonator volume. The corresponding fluctuations in the frequency ω of the $LiNbO_3$ resonator are derived from the equations

$$\omega^2 = \omega_o^2 - 2\Gamma^2 , \qquad \omega\delta\omega = -2\Gamma\delta\Gamma; \qquad S_{\delta\omega/\omega}(f) = (1 / 4Q^4)S_{\delta\Gamma/\Gamma}(f), \tag{47}$$

in which ω_o would be the natural frequency of the unloaded $LiNbO_3$ resonator mode in the absence of the dissipation Γ.

From Eq. (47) the corresponding resonance frequency fluctuations of the $LiNbO_3$ are given for $V\le\varepsilon$ by

$$\omega^{-2}S_{\omega}(f) = (1/4Q^4)(\Lambda/f) = N\alpha\hbar<\omega>/12n\pi mf(cg)^2 Q^4 ; \tag{48}$$

where $Q = \omega/2\Gamma$ is the quality factor of the single-mode resonator considered, and $<\omega>$ is not the circular frequency of the main resonator mode, but rather the practically constant frequency of the average interacting (thermal) phonon. Indeed, there are an average $n_\omega = kT/\hbar\omega$ phonons present in any mode of frequency ω. For the case of $LiNbO_3$ resonators we have used the interacting thermal mode of average frequency $<\omega>$ to calculate the quantum 1/f effect. The corresponding $\Delta\dot{\mathbf{P}}$ in the main resonator mode of frequency ω_0 has to be also included, but is negligible because of the very large number n of phonons present in the main resonator mode and entering in the denominator of Eqs. (46), (48).

Considering the independence of the small ε^3 coherence volumes, Eq. (9) can be written in general in the form

$$S(f) = \beta' V/fQ^4, \text{ for } V \leq \varepsilon, \qquad (49)$$

and

$$S(f) = \beta' \varepsilon^6/fVQ^4, \text{ for } V \geq \varepsilon, \qquad (50)$$

where, with an intermediary value $<\omega>=10^8$/s, with $n=kT/\hbar<\omega>$, T=300K and $kT=4 \cdot 10^{-21}$ J

$$\beta' =(N/V)\alpha\hbar<\omega>/12n\pi g^2 mc^2 =10^{22}(1/137)(10^{-27} 10^8)^2/12kT\pi 10^{-27}\, 9\, 10^{20} \approx 1. \qquad (51)$$

Consequently, the resonance frequency and the optical modulation frequency show quantum 1/f noise by Eqs. (48)-(51).

9. CONCLUSIONS

The following conclusions can be drawn from our results:

-- Quantum 1/f noise modulates the electro-optic parameters, the resonance frequencies, the dark current and other important quantum-optic observables, as we show here in Sec. 8 for $LiNbO_3$..

-- The quantum 1/f effect appears to generate entropy, but this is only because we do not observe the negative conditional entropy contribution of the emitted soft bremsstrahlung photons, or of the oscillator states present in the photon cloud of a physical charged particle. The latter applies to the coherent Q1/fE.

-- The quantum 1/f effect contains Planck's constant in the denominator and diverges in the classical limit. This is connected with the fact that the Q1/fE is observable in the low frequency limit, while other quantum effects appear at high frequencies.

-- The quantum 1/f effect represents a new aspect of quantum physics. Indeed, it requires the notion of physical cross sections and process rates whose quantum expectation values yield the usual definition of cross sections and process rates. The physical quantities contain in addition the macroscopic quantum fluctuations known as quantum 1/f fluctuations.

-- The quantum 1/f effect is a form of quantum chaos, arising only from spontaneous bremsstrahlung.

-- The quantum /f effect limits most high-technology applications and devices. Its knowledge allows us to optimize[12-15] materials, devices and systems, based on the simple quantum 1/f formulas introduced earlier[2, 19].

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