

Quantum 1/f optimization of quantum sensing in spintronic, electro-optic and nano-devices

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ABSTRACT

Most quantum sensing is limited by fundamental 1/f noise, and is described by the author's quantum 1/f noise theory. The present review paper summarizes this new aspect of quantum mechanics and displays its applications in spin valves and related devices, in the classical forms of infrared detectors, and in various other nanodevices, such as MEMs.

In spintronic devices, the quantum 1/f effect causes macroscopic quantum fluctuations of the electronic spin flip rate. They translate into 1/f noise in the leakage current, given by the conventional quantum 1/f formula.

In infrared detectors, the dark current exhibits similar quantum 1/f fluctuations that are present in the elementary cross sections and process rates that limit and determine the dark current. In n^+ -p junction infrared detectors, for instance, at low temperatures, these are mainly the recombination cross sections present in the space charge region and on the surface in the junction region. At higher temperatures, they are mainly the scattering cross sections in the diffusion tail in the p region. Conventional quantum 1/f effect in these scattering cross sections will be noticed as mobility fluctuations, and therefore also as 1/f fluctuations in the diffusion constant, causing again 1/f noise in the dark current

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1. INTRODUCTION

Fluctuations with a spectral density proportional to 1/f are found in a large number of systems in science, technology and everyday life. These fluctuations are known as 1/f noise in general. They have first been noticed in man's earliest amplifiers, have limited the performance of vacuum tubes in the thirties and forties, and have later hampered the introduction of semiconductor devices in the 20th century.

1/f noise is known to limit most high-technology applications, being present both in the low-frequency domain, and at high frequencies, where it manifests itself as phase noise. This ubiquity of 1/f noise has led to many speculations about its nature, since its discovery by Johnson in 1925. Many felt it ought to be something very fundamental, perhaps basic mathematics that could explain the universal 1/f spectrum. The author's universal "Sufficient Criterion" shows that nonlinearity and a certain type of homogeneity lead to a 1/f spectrum in any system described by arbitrarily complicated systems of nonlinear integro-differential equations. In particular, quantum electrodynamics satisfies this criterion at low frequencies. The resulting conventional quantum 1/f effect is a fundamental property of all quantum mechanical physical cross sections, and the coherent quantum 1/f effect is a property of all currents. Together, they yield most of the observed 1/f noise.

For sensing, imaging and remote sensing applications, both baseband 1/f noise and phase noise must be minimized. Using the quantum 1/f theory, this can be done as part of the optimization process on the basis of a properly chosen figure of merit. For this, the quantum 1/f formulas provide an analytical 1/f noise expression of the devices phase noise expression of the system as a function of the device parameters and operation points. The quantum 1/f theory is used to calculate from first principles the 1/f noise present in the device parameters and in the resulting system frequency from various infrared detectors or photodetectors, quartz sensors and resonators, silicon MEMS sensors and resonators, resonant tunneling diodes (RTDs), super-electronic lattice devices (SLEDs), Gunn devices (TEDs), transit time diodes, HFETs, HBTs, etc. In general, quantum 1/f fluctuations in the dissipative elements lead to a Q^4 dependence of the spectral density of fractional frequency fluctuations and of the corresponding phase noise, where Q is the quality factor.

A companion paper is focused on a more rigorous derivation of both forms of the fundamental quantum 1/f noise and applications to LiNbO_3 . Here we start with an elementary physical derivation of the conventional quantum 1/f effect and its relation to the coherent quantum 1/f effect and the resulting quantum 1/f noise. We then apply it to

spintronics, to n⁺ p and MIS infrared detectors, to resonant tunneling diodes, and to ultra-thin semiconductor slabs that can be easily bent.

Elementary derivation of the conventional quantum 1/f effect

This effect¹⁻⁶ is present in any cross section or process rate involving charged particles or current carriers. The physical origin of quantum 1/f noise is easy to understand. Consider for example Coulomb scattering of current carriers, e.g., electrons on a center of force. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by DeBroglie waves of a frequency corresponding to their energy. However, some of the electrons have lost energy in the scattering process, due to the emission of bremsstrahlung. Therefore, part of the outgoing DeBroglie waves is shifted to slightly lower frequencies. When we calculate the probability density in the scattered beam, we obtain also cross terms, linear both in the part scattered with and without bremsstrahlung. These cross terms oscillate with the same frequency as the frequency of the emitted bremsstrahlung photons. The emission of photons at all frequencies results therefore in probability density fluctuations at all frequencies. The corresponding current density fluctuations are obtained by multiplying the probability density fluctuations by the velocity of the scattered current carriers. Finally, these current fluctuations present in the scattered beam will be noticed at the detector as low frequency current fluctuations, and will be interpreted as fundamental cross section fluctuations in the scattering cross section of the scatterer. While incoming carriers may have been Poisson distributed, the scattered beam will exhibit super-Poissonian statistics, or bunching, due to this new effect which we may call quantum 1/f effect. The quantum 1/f effect is thus a many-body or collective effect, at least a two-particle effect, best described through the two-particle wave function and two-particle correlation function.

Let us estimate the magnitude of the quantum 1/f effect semiclassically by starting with the classical (Larmor) formula $2q^2a^2/3c^3$ for the power radiated by a particle of charge q and acceleration a . The acceleration can be approximated by a delta function $a(t) = \Delta v \delta(t)$ whose Fourier transform Δv is constant and is the change in the velocity vector of the particle during the almost instantaneous scattering process. The one-sided spectral density of the emitted bremsstrahlung power $4q^2(\Delta v)^2/3c^3$ is therefore also constant. The number $4q^2(\Delta v)^2/3hfc^3$ of emitted photons per unit frequency interval is obtained by dividing with the energy hf of one photon. The probability amplitude of photon emission $[4q^2(\Delta v)^2/3hfc^3]^{1/2}e^{i\gamma}$ is given by the square root of this photon number spectrum, including also a phase factor $e^{i\gamma}$. Let ψ be a representative Schrödinger catalogue wave function of the scattered outgoing charged particles, which is a single-particle function, normalized to the actual scattered particle concentration. The beat term in the probability density $\rho = |\psi|^2$ is linear both in this bremsstrahlung amplitude and in the non-bremsstrahlung amplitude. Its spectral density will therefore be given by the product of the squared probability amplitude of photon emission (proportional to 1/f) with the squared non-bremsstrahlung amplitude which is independent of f . The resulting spectral density of fractional probability density fluctuations is obtained by dividing with $|\psi|^4$ and is therefore

$$|\psi|^{-4} S_{|\psi|^2}(f) = 8q^2(\Delta v)^2/3hfc^3 = 2\alpha A/fN = j^{-2} S_j(f), \quad (1.1)$$

where $\alpha = e^2/\hbar c = 1/137$ is the fine structure constant and $\alpha A = 4q^2(\Delta v)^2/3hc^3$ is known as the infrared exponent in quantum field theory, and is known as quantum 1/f noise co-efficient, or Hooge constant, in electrophysics.

The spectral density of current density fluctuations is obtained by multiplying the probability density fluctuation spectrum with the squared velocity of the outgoing particles. When we calculate the spectral density of fractional fluctuations in the scattered current j , the outgoing velocity simplifies, and therefore Eq. (1.1) also gives the spectrum of current fluctuations $S_j(f)$, as indicated above. The quantum 1/f noise contribution of each carrier is independent, and therefore the quantum 1/f noise from N carriers is N times larger. However, the current j will also be N times larger, and therefore in Eq. (1.1) a factor N was included in the denominator for the case in which the cross section fluctuation is observed on N carriers simultaneously.

The fundamental fluctuations of cross sections and process rates are reflected in various kinetic coefficients in condensed matter, such as the mobility μ and the diffusion constant D , the surface and bulk recombination speeds s , and recombination times τ , the rate of tunneling j_t and the thermal diffusivity in semiconductors. Therefore, the spectral density of fractional fluctuations in all these coefficients is given also by Eq. (1.1).

When we apply Eq. (1.1) to a certain device, we first need to find out which are the cross sections σ or process rates which limit the current I through the device, or which determine any other device parameter P . Second, we have to determine both the velocity change Δv of the scattered carriers and the number N of carriers simultaneously used to test each of these cross sections or rates. Then Eq. (1.1) provides the spectral density of quantum $1/f$ cross section or rate fluctuations. These spectral densities are multiplied by the squared partial derivative $(\partial I / \partial \sigma)^2$ of the current, or of the device parameter P of interest, to obtain the spectral density of fractional device noise contributions from the cross sections and rates considered. After doing this with all cross sections and process rates, we add the results and bring (factor out) the fine structure constant α as a common factor in front. This yields excellent agreement with the experiment in a large variety of samples, devices and physical systems.

Eq. (1.1) was derived in second quantization, using the commutation rules for boson field operators. For fermions one repeats the calculation replacing in the derivation the commutators of field operators by anticommutators, which yields¹⁻⁶

$$\rho^{-2} S_{\rho}(f) = j^{-2} S_j(f) = \sigma^{-2} S_{\sigma}(f) = 2\alpha A / f(N-1) \quad (1.2)$$

This causes no difficulties, since $N \geq 2$ for particle correlations to be defined, and is practically the same as Eq. (1.1), since usually $N \gg 1$. Eqs. (1.1) and (1.2) suggest a new notion of physical cross sections and process rates which contain $1/f$ noise, and express a fundamental law of physics, important in most high-technology applications.

Coherent Quantum $1/f$ Effect

This effect arises in any electric current in matter (semiconductors, metals, liquids, gases) or in vacuum from the definition of the physical electron (or any other charged particle) as a bare particle plus a coherent state of the electromagnetic field^{11, 17-20}. It is caused by the energy spread, or uncertainty, characterizing any coherent state of the electromagnetic field oscillators. This energy spread, is required by Heisenberg's uncertainty principle, due to the presence of complementary phase information in any coherent state of the field oscillators. This energy spread which spells non-stationarity, i.e., fluctuations, that turn out to be $1/f$ both in the probability density of the electrons and in the current transported by them. Both the conventional quantum $1/f$ effect present in any electron independent of all the others, and the $1/f$ spectrum of the emitted bremsstrahlung photons in a scattering process can be traced to this coherent state of the field of the electron. They are thus both traced to the uncertainty principle.

The $1/f$ spectral density of these coherent quantum $1/f$ fluctuations was obtained both through rigorous derivation from the quantum-electrodynamical propagator of a physical charged particle, and through an earlier elementary derivation given in a companion paper in this volume. The coherent quantum $1/f$ effect is thus essentially caused by the uncertainty present the electron mass caused by the mass contribution from the field of the electron. The spectral density $S_{|\psi|^2}$ for the particle concentration $|\psi|^2$, or S_j for the current density $j = e(k/m)|\psi|^2$ is

$$S_{|\psi|^2} < |\psi|^{-4} > = S_j < j >^{-2} = 2(\alpha / \pi f N) = 4.6 \cdot 10^{-3} f^{-1} N^{-1}. \quad (1.3)$$

Here we have included the total number N of charged particles which are observed simultaneously in the denominator, because the noise contributions from each particle are independent. This result is related to the conventional Quantum $1/f$ Effect introduced above, and was derived directly (Handel^{11, 17-20}) from the physical QED propagator of Kulish, Faddeev and Zwanziger.

Relation to the conventional Quantum $1/f$ Effect.

We turn now to the connection between the conventional and coherent Quantum $1/f$ Effects. The coherent state has an uncertain energy. The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy E_m , per unit length, of the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int (B^2 / 8\pi) d^3x = [nevS/c]^2 \ln(R/r) \quad (1.4)$$

Fig. 1: To define the parameter s , a slice as thick as the classical electron radius is considered. The number of carriers in it is s .

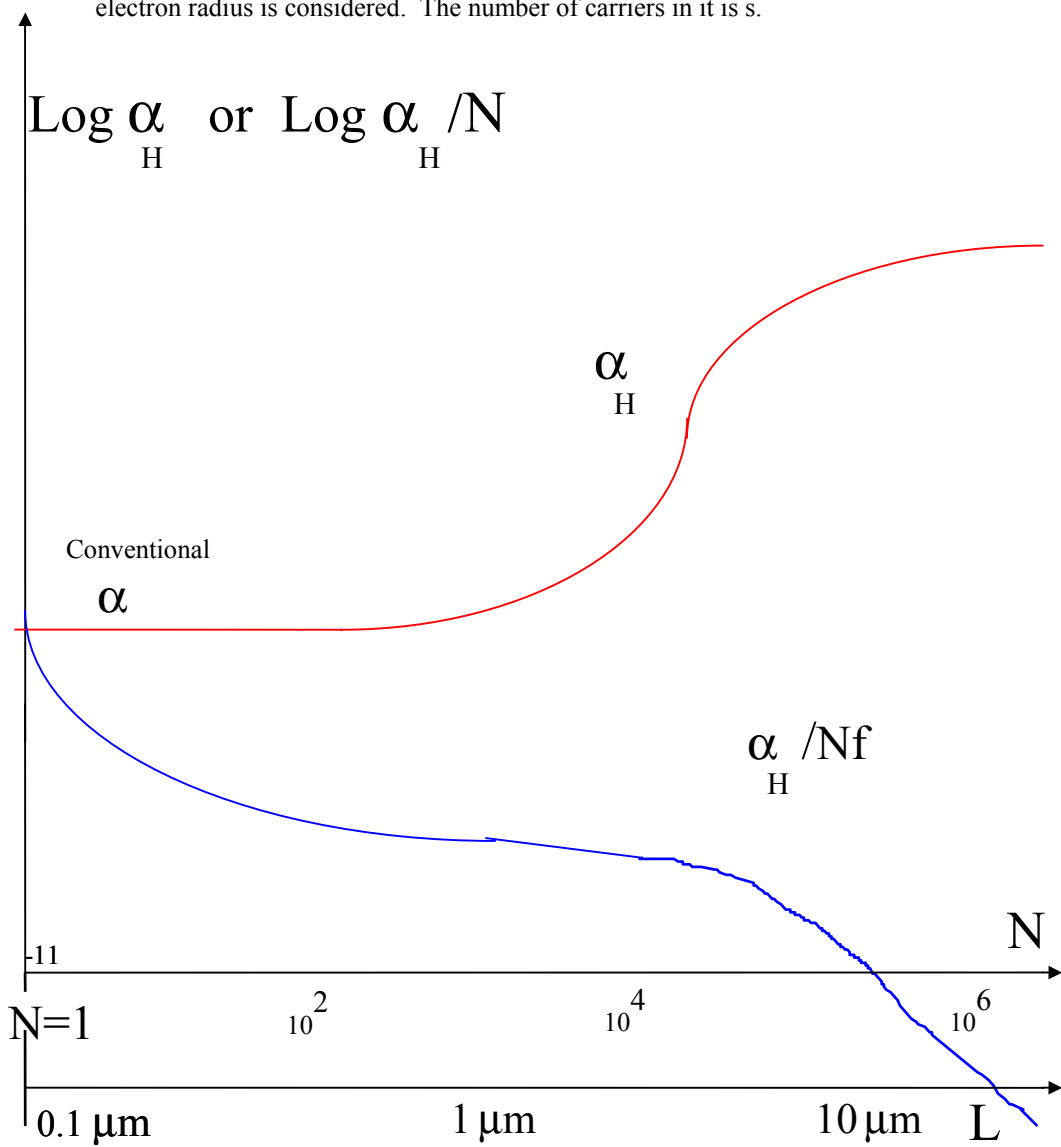


Fig. 2: The quantum 1/f parameter α_H and the resulting spectral density $S_j = \alpha_H / Nf$ as a function of the number of carriers in the sample N or of the cross section size L

is much smaller than the total kinetic energy E_k of the drift motion of the individual carriers

$$E_k = \sum_i m_i v^2 / 2 = n S m v^2 / 2 = E_m / s. \quad (1.5)$$

Here we have introduced the magnetic field B , the carrier concentration n , the cross sectional area S and radius r of the cylindrical sample (e.g., a current carrying wire), the radius R of the electric circuit, and the "coherence ratio"

$$s = E_m / E_k = 2 n e^2 S / m c^2 \ln(R/r) = 2 e^2 N' / m c^2, \quad (1.6)$$

where $N' = nS$ is the number of carriers per unit length of the sample and the natural logarithm $\ln(R/r)$ has been approximated by one in the last form. We expect the observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2) S_\mu(f) = [1/(1+s)] [2\alpha A / f N] + [s/(1+s)] [2\alpha / \pi f N] \quad (1.7)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side. In this equation $\alpha A = 2\alpha (\Delta v/c)^2 / 3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum $1/f$ effect^{1-6, 17-20}. This equation is limited to quantum $1/f$ mobility (or diffusion) fluctuations, and does not include the quantum $1/f$ noise in the surface and bulk recombination cross sections, in the surface and bulk trapping centers, in tunneling and injection processes, in emission or in transitions between two solids.

Note that the coherence ratio s introduced here equals the unity for the critical value $N' = N'' = 2 \cdot 10^{12} / \text{cm}$, e.g. for a cross section $S = 2 \cdot 10^{-4} \text{ cm}^2$ of the sample when $n = 10^{16}$. For small samples with $N' \ll N''$ only the first term survives, while for $N' > N''$ the second term in Eq. (1.7) is dominant.

2. QUANTUM 1/f EFFECT IN SPIN-POLARIZED TRANSPORT

The spin-polarized leakage current passing through a closed spin-valve is caused by electron spin-flip processes. The rate of these processes controls the rate of current flow. Without a spin-flip, the electron can't go through the valve. The electronic spin-flip rate is affected by quantum $1/f$ noise because the spin-flip emits bremsstrahlung just like any sudden current change $\Delta \mathbf{J} = (e/L) \Delta \mathbf{v}$ would do. The conventional quantum $1/f$ effect ($Q1/fE$) in any rate Γ or cross section σ is a macroscopic quantum phenomenon described by the quantum $1/f$ engineering formula,

$$S \delta \Gamma / \Gamma(f) = S \delta \sigma / \sigma(f) = S \delta j / j(f) = (4\alpha / 3\pi f N) (\Delta v/c)^2. \quad (2.1)$$

Here $S \delta \Gamma / \Gamma(f)$ is the spectral density of fractional fluctuations in current, $\delta j / j$, in scattering or recombination cross section $\delta \sigma / \sigma$, or in any other process rate $\delta \Gamma / \Gamma$. $\alpha = e^2 / \hbar c = 1/137$ is Sommerfeld's fine structure constant, a magic number of our world depending only on Planck's constant $\hbar$, the charge of the electron e and the speed of light in vacuum c . $A = 2(\Delta v/c)^2 / 3\pi$ is essentially the square of the vector velocity change Δv of the scattered particles in the scattering process whose fluctuations we are considering, in units of c . Finally, $N = LN'$ is the number of particles used to define the notion of current j , of cross section σ or of process rate Γ .

This applies to the *quantum 1/f fluctuations of the electronic spin-flip rate*, or *decoherence rate*, as mentioned above. Decoherence is caused by the elementary spin-flip of a nucleus due to its various interactions. The particular nature of these is not important here, due to the well-known quantum $1/f$ universality that characterizes all infrared divergence phenomena. The rate of this process has quantum fluctuations according to Eq. (2.1). It reduces the total magnetic moment M of the sample by the magnetic moment $\mu = e\hbar / 2mc = 1$ Bohr magneton of a single electron spin

undergoing a change of one $\hbar$ in its spin projection. The current change $e\Delta v$ causing bremsstrahlung is here $(\dot{\Delta \mathbf{M}})/e$, the change in the rate of demagnetization caused by the emission of an energy quantum, a photon. This yields

$$S_{\delta\Gamma/\Gamma}(f) = 4\alpha\langle(\Delta\mathbf{v})^2\rangle/3\pi f c^2 = 4\alpha\langle(\Delta\dot{\mathbf{M}})^2\rangle/3\pi f e^2 c^2. \quad (2.2)$$

Let N be the number of elementary magnetic dipoles $\mu = e\hbar/2mc$ present in the magnetic induction field B and aligned parallel to it. Applying a variation $\Delta N = 2$ for a spin reversal, we get

$$\Delta n/n = \Delta N/N = |\Delta\dot{\mathbf{M}}|/|\dot{\mathbf{M}}|, \text{ or } \langle(\Delta\dot{\mathbf{M}})^2\rangle = \langle(\dot{\mathbf{M}})^2\rangle/N^2 = 8\mu^4 H^2 N^2/\hbar^2, \quad (2.3)$$

where we used Larmor's theorem $\dot{\mathbf{M}} = \boldsymbol{\omega} \times \mathbf{M}$, with $\boldsymbol{\omega} = N\mu\mathbf{H}/\hbar$. Substituting $\Delta\dot{\mathbf{M}}$ into Eq. (2.2), we get

$$S_{\delta j/j} = \Gamma^{-2} S_{\Gamma}(f) = 32\alpha\mu^4 H^2 N^2/3\pi f e^2 c^2 \hbar^2. \quad (2.4)$$

This is the spectral density of fractional quantum $1/f$ fluctuations in the rate Γ of spin-flip or decoherence (electrodynamical $Q/1fE$ only). This spectral density of fractional fluctuations will also be applicable to the spin-polarized leakage current j through the spin valve. There is also shot noise $S_{\delta j/j} = 2ej$.

3. QUANTUM $1/f$ NOISE IN n^+p JUNCTION AND MIS DIODE INFRARED DETECTORS

As we have seen in the preceding sections, quantum $1/f$ noise is a low-frequency fluctuation process present in elementary cross sections σ and process rates Γ , given by the fractional spectral density

$$S_{\sigma}(f)/\sigma^2 = S_{\Gamma}(f)/\Gamma^2 = (4\alpha/3\pi f N)(\Delta\mathbf{v}/c)^2 \quad (3.1)$$

for conventional quantum $1/f$ noise which is applicable to all devices, but is seen in small devices, and

$$S_{\sigma}(f)/\sigma^2 = S_{\Gamma}(f)/\Gamma^2 = (2\alpha/\pi f N) = \alpha_c/fN \quad (3.2)$$

for coherent state quantum $1/f$ noise which is larger, and applicable to large devices in which the kinetic energy of the carrier drift motion is predominantly magnetic, rather than mechanical. Here $\Delta\mathbf{v}$ is the velocity change of the carriers in the processes considered, $\alpha = 1/137$ is the fine structure constant, N the number of carriers simultaneously interrogating the cross sections or process rates, $\alpha_c = 4.6 \cdot 10^{-3}$ the coherent quantum $1/f$ noise coefficient, and c the speed of light. The two forms of quantum $1/f$ noise are closely related infrared divergence phenomena which arise due to the interaction of electrons and soft photons.

Both in n^+p diodes^{13,15} and metal-insulator-semiconductor⁹ (MIS) devices the current will be determined by cross sections σ_i such as recombination and scattering cross sections (by phonons and lattice defects), or by other process rates (e.g., band to band and trap-assisted tunneling, particularly important in long-wavelength $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ detectors). The spectral density of the resulting quantum $1/f$ current fluctuations is therefore

$$S_I(f) = \sum_i [\partial I/\partial \sigma_i]^2 S_{\sigma_i}(f) + \sum_i [\partial I/\partial \Gamma_i]^2 S_{\Gamma_i}(f), \quad (3.3)$$

where the spectral densities on the right hand side will be given by Eqs. (3.1) or (3.2), depending on the size of the device.

N^+P Junctions

The current density I of Eq. (3.3) contains a diffusion term I_d , a term I_r caused by recombination in the space charge region, a surface recombination term I_s , a tunneling term I_t and a photovoltaic term caused by the creation of electron hole pairs by photons:

$$\begin{aligned} I &= I_d + I_r + I_s + I_t + q\eta\Phi \\ &= qn_i \{ (n_i/n_0)(D_n/\tau_n)^{1/2} (e^{qV/kT} - 1) + (W/\tau) (e^{qV/2kT} - 1) + s \} + I_t + q\eta\Phi. \end{aligned} \quad (3.4)$$

Here n_i is the intrinsic concentration, n_0 the concentration of acceptors on the p side, D_n and τ_n the diffusion constant and lifetime of minority carriers on the p side. W is the width of the depletion region, $\tau = \tau_{po} + \tau_{no}$ the Shockley-Hall-Read lifetime, V the applied voltage, s the surface recombination speed, η the quantum efficiency and Φ the incident flux of photons. With the exception of the last term, the terms in Eq. (3.4) are known as dark current components.

The first term in rectangular brackets in Eq. (3.4) gives the diffusion current density I_d , and yields a noise term^{5,6}

$$S_{Id}(f) = (\alpha_d + \alpha_r)(qI_d/f\tau_n)F(a); \quad \alpha_d = (4\alpha/3\pi)(h/m^*bc)^2 \exp(-\theta/2T) \quad (3.5)$$

in Eq. (3.3) for small devices, and

$$S_{Id}(f) = \alpha_c(qI_d/f\tau_n)F(a) \quad (3.6)$$

for large devices, with $F(a) = -1/3 - 1/2a + 1/a^2 - (1/a^3)\ln(1+a)$ and $a = \exp(qV/kT) - 1$. Here b is the lattice constant, m^* the effective mass of the electrons, and θ the Debye temperature. These results are obtained by adding the scattering and recombination quantum $1/f$ cross section fluctuation spectra for all points on the p side of a long n^+p diode with the appropriate Green function weight $[\partial I/\partial \sigma_i]^2$, as prescribed by Eq. (3.3). The spatial dependence $\exp(-x/L_d)$ of this Green function is given by the diffusion equation applied to electrons on the p side, with L_d being the electron diffusion length. If the length w_p of the p side is shorter than L_d , we have to replace $F(a)$ by $F(a) - F[a \exp(-w_p/L_d)]$ in Eqs. (3.5) - (3.6), and also to include the quantum $1/f$ noise of the recombination cross sections at the p contact. Similarly we obtain⁶

$$\begin{aligned} S_{Ir} &= \alpha_r q I_r \tanh(qV/2kT)/f\tau; \\ &= (4\alpha/3\pi c^2)[2q(V_{diff} - V) + 3kT]\{(m_n^*)^{1/2} + (m_p^*)^{1/2}\}^{-2}. \end{aligned} \quad (3.7)$$

The corresponding expression for the surface recombination current is given by Eq. (3.7) with half of the surface potential jump U added to the diffusion potential V_{diff} . The tunneling current I_t and its noise is the same as in MIS devices; it will be discussed below. In the final form of Eq. (3.3) the fine structure constant α appears as a general factor.

MIS Devices

The MIS infrared detector devices⁹ considered here are homogeneous $Hg_{1-x}Cd_xTe$ slabs of thickness of the order of the minority carrier diffusion length, on which insulated metallic field effect plates are applied with a potential that creates a surface inversion layer. The transition between the bulk and the inverted surface layer is a pn junction which is similar to a conventional pn junction caused by inhomogeneous doping, connected in series with a capacitor, but is free of the defects introduced in the doping process. The theory summarized for junctions in the previous section is again applicable for the quantum $1/f$ noise of the (dark) current flowing through the junction, but this time the longer life times in the denominator of Eqs. (3.5), (3.6) and (3.7) will lead to lower quantum $1/f$ noise. This is a considerable advantage of MIS devices. On the other hand, they have to be operated in a pulsed mode in which the photoelectrically generated carriers separated by the field of the induced junction, and accumulated in the potential well of the surface inversion layer, have to be removed and registered periodically. This is done by alternating the inverting voltage pulses at the plate with short "readout" potentials which generate temporary conditions close to flat energy bands at the surface.

In addition to the photoelectrically generated carriers, the readout signal also contains the carriers accumulated through the various dark current mechanisms mentioned above. Therefore, at low frequencies the $1/f$ noise contained in the dark current I will limit the performance of MIS detectors as infrared detectors.

The integration implied by the pulsed mode of operation acts as a low pass filter which does not influence the signal to noise ratio at frequencies well below the readout frequency. Furthermore, the pulsed mode of operation does not affect the quantum $1/f$ noise, because the quantum $1/f$ fluctuations of cross sections and process rates are continuously tested by carriers in thermal agitation, and are present independently of the method of sampling used by us.

In general the dark current I can again be written in the form

$$I = I_d + I_r + I_s + I_t + q\eta\Phi. \quad (3.8)$$

The five terms on the right hand side correspond to minority carrier diffusion from the bulk, generation from Shockley-Read centers in the depletion region, generation from SR centers at the surface, tunneling, and photoelectric generation by the thermal radiation background flux Φ . We shall give the formulae¹⁰ which determine these terms below, also including an example of their calculation in a p-type device

$$I_d = (qn_i^2/p_o)[kT\mu_n/q\tau_n]^{1/2} = (1.6 \cdot 10^{-19} \text{C} \cdot 36 \cdot 10^{24} \text{cm}^{-6}/10^{15} \text{cm}^3) \times [0.01 \text{V} \cdot 1.5 \cdot 10^5 (\text{cm}^2/\text{Vs})/10^{-6} \text{s}]^{1/2} = 2 \cdot 10^{-4} \text{A/cm}^2, \quad (3.9)$$

$$I_r = qn_i W/2\tau_o = [(1.6 \cdot 10^{-19} \text{C} \cdot 6 \cdot 10^{12} \text{cm}^{-3} \cdot 2 \cdot 10^{-4} \text{cm})/(2 \cdot 10^{-6} \text{s})] = 10^{-4} \text{A/cm}^2 \quad (3.10)$$

$$I_s = J_b = qn_i s/2 = [(1.6 \cdot 10^{-19} \text{C} \cdot 6 \cdot 10^{12} \text{cm}^{-3} \cdot 20 \text{cm/s})] = 1.8 \cdot 10^{-5} \text{A/cm}^2, \quad (3.11)$$

$$I_t = 10^{-13} (N_r \Phi_s / E_g) \exp[-5.3 \cdot 10^6 E_g^2 / E] = 10^{-13} (4 \cdot 10^{12} \text{cm}^{-3} \cdot 0.2 \text{V} / 0.062 \text{V}) \times \exp[-5.3 \cdot 10^6 (0.062 \text{V})^2 / 3000 \text{V/cm}] = 8 \cdot 10^{-4} \text{A/cm}^2, \quad (3.12)$$

Here $n_i = 6 \cdot 10^{12} \text{cm}^{-3}$ is the intrinsic carrier concentration, and the concentration of holes was taken to be $p_o = 10^{15} \text{cm}^{-3}$. The mobility $\mu = 1.5 \cdot 10^5 \text{cm}^2/\text{Vs}$ as well as the life time $\tau_n = 10^{-6} \text{s}$ of the minority carriers have to be replaced by μ_p and τ_p for the case of n-type devices. The surface recombination speed was denoted by $s = 40 \text{cm/s}$, and the concentration of intermediary states effective for tunneling was denoted by $N_r = 4 \cdot 10^{12} \text{cm}^{-3}$. The bandgap considered was $E_g = 0.065 \text{eV}$, the surface potential $\Phi_s = 0.2 \text{V}$ and the electric field below the surface $E = 3000 \text{V/cm}$. All numerical values have been included only as an example and are not characteristic of a particular device. The numerical factors in Eq. (3.12) correspond to p - type HgCdTe with a $20 \mu\text{m}$ cutoff wavelength and were taken from the paper of Kinch and Beck¹⁰. For n-type devices we also need to include tunneling via surface states. If the density of fast surface states is denoted by $N_{fs} = 10^{12} \text{cm}^{-2} \text{V}^{-1}$, the current generated from tunneling via a uniform density of fast surface states across the bandgap will be given by⁹

$$I_{tsc} = -qp_i N_{fs} kT/2q = 1.75 \cdot 10^{-4} \text{A/cm}^2. \quad (3.13)$$

The sum of the first four currents on the right hand side of Eq. (3.8) must be smaller than the fifth which corresponds to the thermal radiation background flux, for background limited (BLIP) performance. Although not all terms in the dark current are of importance, we still retain them, because their quantum 1/f noise may be quite significant, even if the corresponding current is negligible. We shall now proceed with the calculation of quantum 1/f noise contributions from all these currents.

The quantum 1/f noise in I_d and I_r is the same as in n^+p diodes, and is given by Eqs. (3.5)-(3.7).

The surface recombination current is a dark current component originating from surface states at the interface between the surface passivation layer and the bulk. The recombination cross sections of these states will exhibit quantum 1/f noise. The quantum 1/f noise coefficient α_H reflects the velocity change of carriers involved in this process

$$\alpha_H = (4\alpha/3\pi)(2/m^* c^2)[3kT/2 + eU/2 + 0.1 \text{eV}], \quad (3.14)$$

where U is the surface potential jump present between the surface passivation layer and the bulk even if no gate voltage is applied, and V is the applied gate voltage which we take with a coefficient less than unity, here for example with a weight of 0.1.

The calculation of surface recombination quantum 1/f noise is similar to the calculation of quantum 1/f noise from recombination in the space charge region. However, in this case the cross sections are not distributed over the width of

the junction, but rather are concentrated at the surface which is characterized by the surface potential Φ_s . Therefore, with $x=eV/2kT$, we can write an expression of the form

$$S_{IS}(f) = \alpha_H e I_s (\tanh x) / f (\tau_{no} + \tau_{po}) \exp(e\Phi_s/kT). \quad (3.15)$$

We note that the fabrication process of MIS structures introduces less bulk defects than the fabrication of photovoltaic devices. Nevertheless, the fabrication of MIS devices introduces some defects in the bulk layer located right under the surface. These defects will manifest themselves through a contribution to indirect tunneling.

Quantum 1/f Noise in the Tunneling Rate. The tunneling component I_t of the dark current is particularly important in narrow band gap material. In the case of tunneling from a surface accumulation layer to the bulk, the velocity change of the carriers will lead us from the thermal velocity of the carrier on one side of the barrier to the thermal velocity of a carrier on the other side of the barrier, if band to band tunneling is considered. If, however, the tunneling goes via intermediate states located in the bandgap, the velocity is zero as long as the carrier is stationary in the intermediate state. We can therefore write the 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi) 6kT/m^* c^2 \quad (3.16)$$

for band to band tunneling, and

$$\alpha_H = (4\alpha/3\pi) 3kT/m^* c^2 \quad (3.17)$$

for tunneling via intermediate states in the bandgap, where we have considered the average squared velocity change two times smaller. The effective mass is the mass of the minority carriers in the bulk material.

For band to band tunneling from a surface inversion layer to the bulk, the velocity change of carriers corresponds to an energy difference of the order of the bandgap E_g plus an energy difference of the order of the thermal energy $3kT/2$, provided we are dealing with deep inversion, as used in practical MIS devices in the pulsed mode of operation. This yields the quantum 1/f coefficient

$$\alpha_H = (4\alpha/3\pi)(E_g + 3kT/2)/m^* c^2 \quad (3.18)$$

for band to band tunneling. For tunneling via intermediate states in the bandgap the corresponding energy difference will be smaller, and therefore we replace E_g by $E_g/2$

$$\alpha_H = (4\alpha/3\pi)(E_g/2 + 3kT/2)/m^* c^2 \quad (3.19)$$

This relation is applicable both if the intermediate states are located in the depletion region or at the surface.

In the last four equations we did not divide by the number of carriers simultaneously involved in the tunneling process, because this number is less than unity for practical tunneling currents. Whenever a cross section or a process rate is tested with one electron or less than one electron at a time, the effective number of electrons in the denominator of the quantum 1/f formula must be replaced by unity. Let us calculate the average number of carriers simultaneously present in the tunneling process at any time. The tunneling process occurs over a distance $d = E_g/eE$, and the speed v of the carriers will be of the order of the thermal speed in the case of an accumulation layer, and of the order of the bandgap energy in the case of a surface inversion layer. Dividing d by v , we obtain a tunneling time $t = 6 \cdot 10^{-13}$ s for accumulation layers and $t = 3 \cdot 10^{-13}$ s for inversion layers. From Eq. (3.12) we know that the tunneling current is of the order of 10^{-3} A/cm². Multiplying this by t , we obtain $3 - 6 \cdot 10^{-16}$ C, i.e., 2000 - 3000 electrons/cm² tunneling simultaneously in a device of 1 cm² area. In a device of dimensions $50 \mu\text{m} \times 50 \mu\text{m} = 2.5 \cdot 10^{-5}$ cm² the average number of carriers in the process of tunneling at any time is therefore 0.05 - 0.075, and this is indeed much less than unity. Nevertheless, if the area of the device exceeds $5 \cdot 10^{-4}$ cm², Eqs. (3.16) - (3.19) require an additional factor $e/tI_t A$ which makes the noise spectral density proportional to I_t and A , rather than to the square of these quantities.

The photoelectric current will reflect the fluctuations in the number of photons arriving from the radiation

background. The quantum efficiency will not exhibit considerable quantum 1/f noise, because the generated carriers will be collected with certainty. Therefore the collection of photoelectrically generated carriers is not controlled by any cross section or process rate affected by considerable quantum 1/f noise. If we neglect the 1/f noise generated in the series resistance of the diode, there should be no photoelectrically generated 1/f noise from a short-circuited diode.

The resulting 1/f noise. Since the various dark current fluctuations with 1/f spectrum are statistically independent, the total 1/f noise is simply obtained by summing all contributions.

Let us now evaluate the spectral density $S'(f)$ of fractional fluctuations in the various dark current noise contributions per cm^2 of transversal detector (or gate) area, including also a numerical example for a MIS infrared detector. For a given detector, this needs to be divided by the area A of the detector to yield the corresponding fractional spectral densities: $S(f) = [S'(f)]/A$. Fractional fluctuations are dimensionless, so $S'(f)$ will have the dimension of a reciprocal frequency, times a squared length unit which simplifies when we divide by the area of the detector at hand. Let S'_{Id} be the spectral density of fractional fluctuations in the noise caused by quantum 1/f fluctuations in diffusion, S'_{Ir} in bulk recombination, S'_{Is} in surface recombination, and S'_{It} in tunneling. With $m_p^* = 0.55 m_0$, $m_n^* = 0.02 m_0$, $\tau_n = 10^{-6} \text{s}$, $E_g = 0.1 \text{ eV}$, $3kT/2 = 0.01 \text{ eV}$, we obtain for a p-type MIS device with $w_p \gg L_d$

$$\begin{aligned} S'_{Id} &= (\alpha_{nd} + \alpha_{nr})[e/f\tau_n I_d]F(a) = \alpha_{coh} \{e^{1/2}/[f(kT\mu\tau_n)^{1/2}N_p]\}F(a)/a \\ &= (4.6 \cdot 10^{-3}/4fN_p) 4 \cdot 10^{-10} C^{1/2}/[10^{-6} \text{s} \cdot 1.5 \cdot 10^5 (\text{cm}^2/\text{Vs}) 4 \cdot 10^{-21} \text{J}]^{1/2} \\ &= 1.8 \cdot 10^{-6} \text{cm}^2/\text{f}, \text{ [or } = 10^{-8} \text{ cm}^2/\text{f, with } \alpha_p = 2 \cdot 10^{-5} \text{ for incoherent noise]} \end{aligned} \quad (3.20)$$

$$\begin{aligned} S'_{Ir} &= [\alpha_{nr} e/f(\tau_{no} + \tau_{po})I_r] \tanh x = [\alpha_{nr} e/(feAwn_i \tanh x)] \tanh x \\ &= \alpha_{nr} e/feAwn_i = 4.6 \cdot 10^{-9} \text{cm}^2/\text{f} \quad (x = eV/2kT) \end{aligned} \quad (3.21)$$

$$\begin{aligned} S'_{Is} &= (4\alpha/3\pi)(2/m^* c^2)[3kT/2 + eU/2 + 0.1 \text{Ve}] [e (\tanh x)/f(\tau_{no} + \tau_{po})I_s] \\ &= (4\alpha/3\pi 0.02)(2/500,000)[0.025 + 0.5 + 0.5] [e (\tanh x)/feAwn_i(e^x - 1)] = 7 \cdot 10^{-8} \text{cm}^2/\text{f} \end{aligned} \quad (3.22)$$

$$S'_{It} = (4\alpha/3\pi)(E_g + 3kT/2)/m^* c^2 = (4/9.5 \cdot 137 \cdot 0.02)(0.11/500,000) = 3.3 \cdot 10^{-8} \text{cm}^2/\text{f} \quad (3.23)$$

S'_{Id} was calculated in the small bias limit for $w_p \gg L$, but $w_p = 0.25 L$ gives the same result; the incoherent case with a lattice constant of $b = 0.65 \text{ nm}$ and $\theta = 320 \text{ K}$ was also listed above, and would give $1.8 \cdot 10^{-10} \text{cm}^2/\text{f}$ for a n-type device. Eqs. (3.20) - (3.23) would be reduced $m_p^*/m_n^* = 27.5$ times for n-type devices. The noise S'_{Is} in the front surface recombination current I_s has been calculated with the inclusion of a term of 10% of the applied gate voltage V ($= 5 \text{ volt}$ in our example) into the kinetic energy of the carriers at the surface. For the back surface recombination current, this term has to be dropped in the similar expression of S'_{ib} . However, we have neglected this here, because the surface recombination terms will not turn out to be important, as we will see below. Using Eq. (3.8) and the current densities evaluated in Eqs. (3.20) – (3.23) to calculate the fraction of each current, we obtain

$$1 \text{ cm}^2 \times S'(f) = (20/132)^2 1.8 \cdot 10^{-6} + (10/132)^2 4.6 \cdot 10^{-9} + (3.6/132)^2 7 \cdot 10^{-8} + (0.01/132)^2 3.3 \cdot 10^{-8} + (80/132)^2 1.8 \cdot 10^{-8} + (17.5/132)^2 1.8 \cdot 10^{-8} = 3.67 \cdot 10^{-8} + 2.6 \cdot 10^{-11} + 5.2 \cdot 10^{-11} + 1.9 \cdot 10^{-16} + 6.61 \cdot 10^{-9} + 3.17 \cdot 10^{-10} = 4.37 \cdot 10^{-8}, \text{ or for coherent 1/f noise, } 7.1 \cdot 10^{-9} \text{ (p-type device); and } 3 \cdot 10^{-10} \text{ (n-type device).} \quad (3.24)$$

The detectivity of infrared detectors is limited in general by three types of noise: (i) current noise in the detector, (ii) noise due to background photons (photon noise), (iii) noise in the electronic system following the detector. We shall neglect here the background photon noise and the noise in the electronic system. The detectivity is defined as

$$D^*(\lambda, f) = (A\Delta f)^{1/2} / \text{NEP}(\text{cmHz}^{1/2}/\text{w}), \quad (3.25)$$

where A is the area of the detector, NEP the noise equivalent power defined as the r.m.s. optical signal power (in w) of wavelength λ required to produce an r.m.s. voltage (current) equal to the r.m.s. noise voltage (current) of the device in a bandwidth Δf , and f is the frequency of modulation. The noise equivalent power NEP is given by

$$\text{NEP} = (h\nu/\eta q)(S_{\text{Id}}(f)\Delta f)^{1/2}. \quad (3.26)$$

Therefore, we obtain for the detectivity

$$D^*(\lambda, f) = (\eta q \lambda / hc) [A/S_{\text{Id}}(f)]^{1/2} = (\eta q \lambda / hc) [S_{\text{Id}}(f)]^{-1/2} \quad (3.27)$$

We notice that $D^*(\lambda, f)$ is proportional to λ up to the peak wavelength λ_c . For $\lambda > \lambda_c$ we have $\eta = 0$ and thus $D^*(\lambda, f) = 0$. By substituting our result for S_{Id} , we obtain the general expression of the detectivity as a function of various parameters of the MIS device. The value calculated in Eq. (3.24) can be used in order to estimate the detectivity of the device in our example. Substituting into Eq. (4.6), we obtain with a quantum efficiency $\eta = 0.7$ and wavelength of $10 \mu\text{m}$

$$D^*(\lambda, f) = (\eta q \alpha / hc) [S_{\text{Id}}(f)]^{-1/2} = [0.7 \cdot 1.6 \cdot 10^{-19} \text{C} \cdot 10^{-5} \text{m} / (6.6 \cdot 10^{-34} \text{Js} \cdot 3 \cdot 10^{-8} \text{m/s})] [f / (4.37 \cdot 10^{-8} \text{cm}^2 \cdot 1.74 \cdot 10^{-6} \text{A}^2/\text{cm}^4)]^{1/2} \\ = 2 \cdot 10^7 (\text{cm Hz}^{1/2}/\text{w}) f^{1/2}, \text{ or for incoherent } 1/f \text{ noise, } 5 \cdot 10^7 (\text{p}); \text{ and } 2.5 \cdot 10^8 (\text{n}). \quad (3.25)$$

In conclusion we note that for the relatively large devices which we have considered, most of the quantum $1/f$ noise comes from fluctuations in diffusion and in the tunneling rate via impurity centers in the bandgap. The effective mass of the carriers is present in the denominator of all quantum $1/f$ noise contributions except the coherent quantum $1/f$ fluctuation present in the diffusion current of large devices. In smaller devices the diffusion current will also be given by the conventional quantum $1/f$ formula which contains the effective mass of the carriers in the denominator. For umklapp scattering the mass of the carriers in the denominator is even squared. Consequently, we expect lower quantum $1/f$ noise from n – type devices, in which the minority carriers are holes, particularly if the devices are small, e.g., below $10 \mu\text{m}$.

4. PHASE NOISE IN RTDs AND IN OSCILLATORS BASED ON THEM

Resonant tunneling diodes have been proposed as generators of THz oscillations and radiation. They consist of two potential barriers enclosing a quantum well. Electrons penetrating the potential barriers by tunneling are controlled by the quasistationary energy levels, defined by the potential well. If their energy is close to the first energy level in the well, resonance occurs, and a peak I_p of the current through the diode occurs. This corresponds to an applied bias voltage V_p . If, however, the applied voltage increases further, only a negligibly small non-resonant current trickle remains at the voltage $V = V_V$ and a broad valley is observed in the I/V characteristic. Scattering processes that reduce the energy of the carriers to a value close to eV_p will always be present, generating a finite current minimum I_V at V_V . Between V_p and V_V there is a negative differential conductance

$$G = -(I_p - I_V) / (V_V - V_p) \quad (4.1)$$

on the I/V curve, that is used to generate oscillations. The $1/f$ noise in I_V is given by Eqs. (1.1)-(1.2) with

$$(\Delta v/c)^2 = 2eV_V/m. \quad (4.2)$$

Taking for instance $V_p = 0.4 \text{ V}$, $I_p = 2.5 \cdot 10^8 \text{ A/m}^2$, $V_V = 0.6 \text{ V}$, $I_V = 4 \cdot 10^7 \text{ A/m}^2$, we obtain with $m_{\text{eff}} = 0.068 m_0$,

$$N I_V^{-2} S_{I_V}(f) = 2\alpha A / f = 7.4 / 0.068 \cdot 10^{-9} = 1.3 \cdot 10^{-7} \quad (4.3)$$

N is given by

$$N = \tau I_V / e, \quad (4.4)$$

where τ is the lifetime of the carriers. With a cross-sectional area of 10^{-6} cm^2 and $\tau = 10^{-10}$, we obtain $N = 2.5 \cdot 10^6$.

Finally, the quantum $1/f$ frequency fluctuations can be obtained from the formula

$$S_{\delta\omega/\omega} = (1/4Q^4)S_{\delta G/G} \quad (4.5)$$

which is derived below at the end of this section. This finally yields with Eq. (1.1)

$$S_{\delta\omega/\omega} = (1/4Q^4)(4\alpha/3\pi)(2eV_v/mc^2) \quad (4.6)$$

for the fractional frequency fluctuation spectrum exhibited by the RTD if included in an RF circuit of quality factor Q . The corresponding phase noise can be obtained from the relations

$$S_{\phi}(f) = (\omega_r/2\pi f)^2 S_y(f), \quad (4.7)$$

or

$$L(f) = (1/2)S_{\phi}(f). \quad (4.8)$$

For the two-sided spectral density $L(f)$.

Finally, we give a simple derivation of the well-known $1/Q^4$ formula derived first in 1978, and widely used today, as we saw also here in this derivation. Resonant systems can best be described as harmonic oscillators with losses

$$dx/dt + \gamma dx/dt + \omega_0^2 x = F(t). \quad (4.9)$$

The quantum $1/f$ fluctuations are present in the loss coefficient γ . They are given by an expression of the form

$$S_{\delta\gamma/\gamma}(f) = \Lambda/f, \quad (4.10)$$

where Λ is a quantum $1/f$ coefficient characterizing the elementary loss process.

The resonance frequency is given by

$$\omega_r^2 = \omega_0^2 + \gamma^2. \quad (4.11)$$

The quantum $1/f$ fluctuations in the resonance frequency are given by $\omega_r \delta\omega_r = -2\gamma\delta\gamma$, or

$$\delta\omega_r/\omega_r = -2(\gamma/\omega_r)^2 \delta\gamma/\gamma = -(1/2Q^2)(\delta\gamma/\gamma), \quad (4.12)$$

where $Q = \omega_r/2\gamma$ is the quality factor. Squaring, averaging and particularizing Eq. (5.16) for the unit frequency interval, we obtain the Q^{-4} law

$$S_y(f) = \langle (\delta\omega_r/\omega_r)^2 \rangle_f = (1/4Q^4) \langle (\delta\gamma/\gamma)^2 \rangle_f = \Lambda/4fQ^4, \quad (4.13)$$

where y is the fractional frequency fluctuation $\delta\omega_r/\omega_r$. This law was first applied to quartz resonators, where it was introduced¹⁴ by the author in 1978.

5. LOW-FREQUENCY NOISE IN BENT ULTRATHIN SEMICONDUCTORS

In ultra-thin semiconductor devices, surface scattering can no longer be ignored and significantly lowers the mobility of the carriers. This contributes a term in the scattering rate, which is proportional to λ/a in first approximation, where a is the thickness of the crystalline semiconductor sample. Since umklapp and intervalley scattering are the main limitation on the mobility of current carriers in silicon, and since they are affected by the largest conventional quantum $1/f$ effect, we expect a $1/f$ noise reduction in properly fabricated ultrathin semiconductor samples.

To estimate the conventional quantum $1/f$ effect ($CQ1/fE$) in ultrathin Si devices and integrated circuits (IC), we need to also include the effect of the reduction in carrier lifetime τ due to the surface recombination rate. This increases

the 1/f noise in junction devices, because the quantum 1/f effect is inversely proportional to the number N of carriers which have participated in the generation-recombination-limited current transport through the various p-n junctions. This number is, however, given by the GR component of the current I multiplied by the lifetime τ of the carriers.

Assuming small samples, the CQ1/fE is applicable and is affecting surface scattering rates according to the fundamental formula

$$S_{\delta\Gamma/\Gamma} = (4\alpha/3\pi) \langle (\Delta v/c)^2 \rangle. \quad (5.1)$$

The brackets indicate a statistical average over the parameters of a surface scattering. We obtain the 1/f noise power spectrum by substituting a considerably increased thermal velocity for Δv , because of the surface potential ($V_0 > 0$ in n-type) in the presence of an accumulation layer, which is not applicable for $V_0 < 0$ (depletion in n-type material). However, the case of inversion would lead again to a 1/f noise increase in spite of $V_0 < 0$, but there is also a carrier identity switch which affects the result in this case, due to the difference in effective mass and scattering mechanism.

Surface scattering and bulk scattering

The total scattering rate is the sum of surface and bulk scattering rates. Therefore, the mobility μ will be determined from the mobility μ_b existent in the absence of surface scattering in the bulk material and from the mobility μ_s which would be present in that sample in the absence of bulk scattering

$$1/\mu = 1/\mu_b + 1/\mu_s. \quad (5.2)$$

The quantum 1/f fluctuations will therefore satisfy the relation

$$\delta\mu/\mu^2 = \delta\mu_b/\mu_b^2 + \delta\mu_s/\mu_s^2. \quad (5.3)$$

Consequently, the CQ1/fE spectral density $S_{\delta\mu/\mu}(f) \equiv \langle (\delta\mu/\mu)^2 \rangle_f$ satisfies the relation

$$S_{\delta\mu/\mu} = (\mu^2/\mu_b^2) \langle (\delta\mu_b/\mu_b)^2 \rangle_f + (\mu^2/\mu_s^2) \langle (\delta\mu_s/\mu_s)^2 \rangle_f. \quad (5.4)$$

For the quantum 1/f noise in the bulk mobility we write

$$\langle (\delta\mu_b/\mu_b)^2 \rangle_f = (4\alpha/3\pi Nf) \langle (\hbar \Delta k/mc)^2 \rangle = (4\alpha/3\pi Nf) (0.75\hbar/amc)^2, \quad (5.5)$$

where m is the effective mass of the carriers, and the effective Δk was taken to be 0.75 G. Here G is the smallest reciprocal lattice vector $2\pi/a$, where a is the lattice constant, and $\hbar$ is Planck's constant, not divided by 2π . Finally, α is Sommerfeld's fine structure constant and N is the number of carriers in the sample.

The conventional Q1/f noise in the surface-scattering-limited mobility μ_s is obtained from Eq. (5.1) by substituting

$$(\Delta v/c)^2 = 6kT/mc^2 + 2eV/mc^2. \quad (5.6)$$

This yields

$$\langle (\delta\mu_s/\mu_s)^2 \rangle_f = (4\alpha/3\pi Nf) [6kT/mc^2 + 2eV_0/mc^2], \quad (5.7)$$

where e is to be replaced by -e, and m by m' the case of inversion, m and m' being the effective masses of the carriers.

The final expression for the resulting power spectrum of quantum 1/f mobility fluctuation is thus

$$S_{\delta\mu/\mu} \equiv \alpha_{\text{conv}} / Nf = (4\alpha/3\pi Nfc^2)[(\mu/\mu_b)^2(0.75h/am)^2 + (\mu/\mu_s)^2(6kT/m + 2eV_0/m)]. \quad (5.8)$$

In this expression the quantity in round brackets in the second term on the right hand side is always smaller than the corresponding round bracket present in the first term, corresponding to lattice scattering, and characteristic for the bulk material. However, the relative contribution of the two terms on the right hand side depends on the sample thickness. For a given sample this second term can be further reduced by treating the surface in order to obtain a depletion layer (with a small $V_0 < 0$ for n-type material).

Influence of dislocations arising from bending deformation

Uniform bending with a radius of curvature R will generate a surface density of dislocations of $\rho = 1/aR$ and an additional volume density of defects $n_{\text{dis}} = 1/a^2R$ in the given ultrathin sample. Assuming we know the measured mobility μ_0 of the given sample before bending, the theoretical mobility $\mu_i = \mu_{\text{lattice}}$ of the ideal sample with no defects (limited by phonon scattering, intervalley and umklapp only) and the concentration of defects n_{odef} in the given sample before bending, we can derive for the quantum 1/f noise spectral density of fractional fluctuations expected in the bent ultra-thin sample the approximate formula

$$S_{\delta\mu/\mu} = \{(\mu_0^2)/[(1+r)\mu_i - r\mu_0]^2\} S_{\delta\mu/\mu}^{\text{latt}}(f), \quad (5.9)$$

in which $r = n_{\text{dis}}/n_{\text{odef}}$, and $S_{\delta\mu/\mu}^{\text{latt}}(f)$ is the theoretical quantum 1/f spectral density affecting μ_i in an ideal sample with no defects, being caused by phonon scattering, intervalley and umklapp only. This formula is obtained by neglecting the quantum 1/f noise of defect scattering (both ionized or neutral, both impurity defects and lattice defects) compared to the much larger quantum 1/f noise of lattice scattering, including phonon scattering, intervalley and umklapp in the ideal lattice.

If a lattice constant of $a = 1 \text{ \AA}$ and a bending radius of curvature $R = 10 \text{ cm}$ are considered, the resulting dislocation densities of $\rho = 10^7 \text{ cm}^{-2}$ and $n_{\text{dis}} = 10^{15} \text{ cm}^{-3}$ have to be compared with the preexisting concentration of defects n_{odef} . Assuming $n_{\text{odef}} = 10^{16} \text{ cm}^{-3}$, we obtain $r = 0.1$. The resulting correction is a small reduction in the expected noise. However, this calculation neglects all current-redistribution effects that result from a non-uniform distribution of dislocations introduced by bending. It also neglects the effect of the decreased lifetime of the carriers in bent ultrathin ICs, in particular on the junction devices or on other unwanted junctions present in the IC. All these effects result in increased conventional quantum 1/f noise. The increased non-uniformity of the current distribution also results in an increase of the coherent quantum 1/f effect present in the larger samples as we see in the next section.

6. CONCLUSIONS

The examples considered in this paper illustrate the application of the quantum 1/f theory to the practical calculation of 1/f noise in various electro-optic devices. Since they provide analytical formulae, they can be used for the optimization of these device for ultra-low 1/f noise. The optimization process starts from a suitably chosen figure of merit, that incorporates the most important objectives along with 1/f noise.

The graph on Fig. 1 shows that after two decades of relatively benign 1/f noise, we are again in trouble with the miniaturization process. Indeed, we are leaving the horizontal part of the graph by entering the nanoscopic domain. The transition from coherent to to conventional quantum 1/f noise is behind us and a steep increase lies ahead of us. Therefore, the application of the quantum 1/f theory and formulas gains additional importance at the present time.

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